TOPIC 1 Scalars

SQ 2.1.1 Why is it necessary to distinguish between scalar and vector quantities?

Reason

Many problems in physics require us to distinguish between scalar and vector quantities in order to apply the correct mathematical and conceptual approaches.

SQ 2.1.2 How does understanding scalars and vectors help us?

Answer

It helps us to grasp how physics applies to real-world situations, such as calculating the total distance travelled (a scalar) or determining the magnitude and direction of a force (a vector).

SQ 2.1.3 With what is chapter 2 primarily concerned?

Answer

It is primarily concerned with vector algebra and its application in uniformly accelerated motion in a straight line, motion of freely falling bodies in a uniform gravitational field, projectile motion, and interaction between objects in one and two dimensions.

SQ 2.1.4 What are scalars?

Definition

Scalars are physical quantities that are described solely by a magnitude, without any mention of direction.

Property

Scalars are directionless and can be fully characterized by a single number and its unit.

SQ 2.1.5 Define mass and distance as scalar quantities.

Mass

The amount of matter in an object, for example $2\,\mathrm{kg}$.

Distance

The total length of the path travelled by an object, irrespective of the direction, for example $50\,\mathrm{m}$.

SQ 2.1.6 Define speed and time as scalar quantities.

Speed

The rate at which an object covers distance, for example $40\,\mathrm{km\,h^{-1}}$.

Time

The duration between two events taking place, for example $20\,\mathrm{s}$.

SQ 2.1.7 Define energy and temperature as scalar quantities.

Energy

The capacity to do work, for example $25\,\mathrm{J}$.

Temperature

A measure of the average kinetic energy of particles in a substance, for example $20\,^{\circ}\mathrm{C}$.

TOPIC 2 Vectors

SQ 2.2.1 What are vectors?

Definition

Those physical quantities which require magnitude as well as direction for their complete specification are known as vectors.

SQ 2.2.2 Differentiate between scalars and vectors.

Scalars	Vectors
They are described solely by a magnitude.	They require magnitude as well as direction.
They are directionless.	They have a specific direction.
Mass, distance, speed and time are examples.	Displacement, velocity, acceleration and force are examples.

SQ 2.2.3 Define displacement and velocity as vector quantities.

Displacement

The change in position of an object, having a magnitude and a direction, for example $14\,\mathrm{m}$ towards west.

Velocity

The speed of an object in a particular direction, for example $50\,\mathrm{km\,h^{-1}}$ towards west.

SQ 2.2.4 Define acceleration and force as vector quantities.

Acceleration

The rate of change of velocity that occurs in either speed or direction or both, for example $10\,\mathrm{m\,s^{-2}}$ upward.

Force

A push or pull acting on an object, determined by its magnitude and direction, for example $20\,\mathrm{N}$ to the right.

SQ 2.2.5 How is a vector represented graphically?

Answer

A vector quantity is represented by a vector diagram in which vectors are shown by arrows. The length of the arrow indicates the magnitude and the head of the arrow shows the direction of the vector.

SQ 2.2.6 How are vectors typically denoted in writing?

Answer

Vectors are typically denoted by bold face letters such as **V** and **F**, or by an arrow above the symbol such as $\vec{A}$.

SQ 2.2.7 What is a component of a vector?

Definition

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions.

SQ 2.2.8 What are rectangular components of a vector?

Definition

It is usually convenient to resolve a vector into its components along mutually perpendicular directions. Such components are called rectangular components.

SQ 2.2.9 Write the vector A in terms of its rectangular components.

Formula

$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$

This is obtained by applying the head to tail rule.

SQ 2.2.10 Write the magnitude of the x-component of a vector A making an angle θ with the x-axis.

Formula

$A_x = A\cos\theta$

SQ 2.2.11 Write the magnitude of the y-component of a vector A making an angle θ with the x-axis.

Formula

$A_y = A\sin\theta$

SQ 2.2.12 How is the magnitude of a vector determined from its rectangular components?

Method

The magnitude is found by using the Pythagorean theorem in the right angled triangle formed by the components.

Formula

$A = \sqrt{A_x^2 + A_y^2}$

SQ 2.2.13 How is the direction of a vector determined from its rectangular components?

Formula

$\tan\theta = \dfrac{A_y}{A_x}$

$\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$

SQ 2.2.14 Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to either of them.

Method

Taking F_1 along the x-axis, the components of the resultant are

$R_x = F_1 + F_2\cos\theta$

and

$R_y = F_2\sin\theta$

Solution

Putting

$R = F_1 = F_2 = F$

in

$R^2 = R_x^2 + R_y^2$

gives

$2\cos\theta + 1 = 0$

Result

$\theta = \cos^{-1}(-0.5) = 120^\circ$

TOPIC 3 Product of Two Vectors

SQ 2.3.1 How many types of vector multiplication are there?

Answer

There are two types of vector multiplications. Their products are known as the scalar product and the vector product.

SQ 2.3.2 Differentiate between scalar product and vector product.

Scalar Product	Vector Product
The product of two vectors results in a scalar quantity.	The product of two vectors results in a vector quantity.
It is written as $\mathbf{A}\cdot\mathbf{B}$.	It is written as $\mathbf{A}\times\mathbf{B}$.
It is commutative.	It is non-commutative.

SQ 2.3.3 Define the scalar or dot product of two vectors.

Definition

The scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is written as $\mathbf{A}\cdot\mathbf{B}$ and is defined as:

Formula

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

Here A and B are the magnitudes of the vectors and θ is the angle between them.

SQ 2.3.4 Give the physical interpretation of the dot product.

Interpretation

The two vectors are first brought to a common origin. Then $\mathbf{A}\cdot\mathbf{B}$ is A multiplied by the projection of B on A, that is the magnitude of the component of $\mathbf{B}$ in the direction of $\mathbf{A}$.

SQ 2.3.5 Express work done as a scalar product.

Explanation

Work done is the effective component of force in the direction of motion multiplied by the distance moved, that is $(F\cos\theta)d$.

Formula

$\mathbf{F}\cdot\mathbf{d} = Fd\cos\theta = \text{Work done}$

SQ 2.3.6 Why is the scalar product commutative?

Reason

Since

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

and

$\mathbf{B}\cdot\mathbf{A} = BA\cos\theta$

therefore

$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$

The order of multiplication is irrelevant.

SQ 2.3.7 What is the scalar product of two mutually perpendicular vectors?

Answer

The scalar product of two mutually perpendicular vectors is zero, because

$\theta = 90^\circ$

Formula

$\mathbf{A}\cdot\mathbf{B} = AB\cos 90^\circ = 0$

SQ 2.3.8 What is the scalar product of two parallel vectors?

Answer

For parallel vectors
$\theta = 0^\circ$
so the scalar product is equal to the product of their magnitudes.

Formula

$\mathbf{A}\cdot\mathbf{B} = AB\cos 0^\circ = AB$

SQ 2.3.9 What is the scalar product of two antiparallel vectors?

Answer

For antiparallel vectors
$\theta = 180^\circ$

Formula

$\mathbf{A}\cdot\mathbf{B} = AB\cos 180^\circ = -AB$

SQ 2.3.10 What is the self product of a vector?

Answer

The self product of a vector is equal to the square of its magnitude.

Formula

$\mathbf{A}\cdot\mathbf{A} = AA\cos 0^\circ = A^2$

SQ 2.3.11 Write the scalar product of two vectors in terms of their rectangular components.

Formula

$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

SQ 2.3.12 How is the angle between two vectors found using the dot product?

Formula

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

SQ 2.3.13 Define the vector or cross product of two vectors.

Definition

The vector product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is a vector defined as:

Formula

$\mathbf{A}\times\mathbf{B} = AB\sin\theta\,\hat{n}$
Here $\hat{n}$ is a unit vector perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$.

SQ 2.3.14 State the right hand rule for the vector product.

Statement

Place together the tails of $\mathbf{A}$ and $\mathbf{B}$ to define their plane. The direction of the product vector is perpendicular to this plane.

Rule

Rotate the first vector $\mathbf{A}$ into $\mathbf{B}$ through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector is along the erect thumb.

SQ 2.3.15 Why is the cross product non-commutative?

Reason

Because of the direction rule, $\mathbf{B}\times\mathbf{A}$ is a vector opposite in sign to $\mathbf{A}\times\mathbf{B}$.

Formula

$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$

SQ 2.3.16 When does the cross product of two vectors have maximum magnitude?

Answer

The cross product of two perpendicular vectors
$(\theta = 90^\circ)$
has maximum magnitude.

Formula

$\mathbf{A}\times\mathbf{B} = AB\sin 90^\circ\hat{n} = AB\,\hat{n}$

SQ 2.3.17 What is the cross product of two parallel or antiparallel vectors?

Answer

The cross product of two parallel or antiparallel vectors is a null vector, because for such vectors
$\theta = 0^\circ$
or 180°.

Consequence

$\mathbf{A}\times\mathbf{A} = 0$

SQ 2.3.18 What does the magnitude of A × B represent geometrically?

Answer

The magnitude of $\mathbf{A}\times\mathbf{B}$ is equal to the area of the parallelogram formed with $\mathbf{A}$ and $\mathbf{B}$ as two adjacent sides.

SQ 2.3.19 Express torque as a vector product.

Explanation

When a force $\mathbf{F}$ is applied on a rigid body at a point whose position vector is $\mathbf{r}$ from any point on the axis of rotation, the turning effect of the force is called torque.

Formula

$\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$

SQ 2.3.20 Express the magnetic force on a moving charge as a vector product.

Explanation

The force on a particle of charge q moving with velocity $\mathbf{v}$ in a magnetic field of strength $\mathbf{B}$ is given by a vector product.

Formula

$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$

SQ 2.3.21 At what angle does the dot product become equal to the cross product?

Answer

The dot product equals the cross product when
$AB\cos\theta = AB\sin\theta$
that is when
$\tan\theta = 1$

Result

$\theta = 45^\circ$

TOPIC 4 Equations of Motions

SQ 2.4.1 What are the three kinematic variables used in the equations of motion?

Answer

The equations of motion describe the motion of an object in terms of its velocity v, position S and time t.

SQ 2.4.2 How are the three equations of motion named?

Answer

The three ways of pairing the kinematic variables are velocity-time, position-time and velocity-position. In this order they are called the first, second and third equation of motion.

SQ 2.4.3 To which objects can the equations of motion be applied?

Condition

The equations of motion can only be applied to those objects which are moving in a straight line with constant acceleration.

SQ 2.4.4 Derive the first equation of motion.

Derivation

The acceleration of a body whose velocity changes from v_i to v_f in time t is

$a = \frac{v_f - v_i}{t}$

Rearranging gives

$v_f - v_i = at$

Result

$v_f = v_i + at$

SQ 2.4.5 What does the first equation of motion correlate?

Answer

It correlates the final velocity attained by a body with its initial velocity and the time interval t, when the body moves with constant acceleration.

SQ 2.4.6 How is the first equation of motion derived graphically?

Derivation

On a velocity-time graph,

$OA = v_i$

and

$DB = v_f$

and

$DB = DC + CB = OA + CB$

so

$v_f = v_i + CB$

The slope of line AB gives the acceleration, so

$a = \frac{CB}{AC}$

and

$CB = at$

Result

$v_f = v_i + at$

SQ 2.4.7 Derive the second equation of motion.

Derivation

Displacement is average velocity multiplied by time, so

$S = \left(\frac{v_i + v_f}{2}\right)t.$

Using the first equation of motion

$v_f = v_i + at$

gives

$S = \left(\frac{v_i + v_i + at}{2}\right)t.$

Result

$S = v_i t + \frac{1}{2}at^2$

SQ 2.4.8 How is the second equation of motion derived graphically?

Derivation

The displacement is the area of the figure $OABD$, which is the area of rectangle $OACD$ plus the area of triangle ABC. This gives

$S = (OA \times OD) + \frac{1}{2}(AC \times BC) = v_i t + \frac{1}{2}(t)(at).$

Result

$S = v_i t + \frac{1}{2}at^2$

SQ 2.4.9 Derive the third equation of motion.

Derivation

From

$2S = (v_i + v_f)t$

and

$t = \frac{v_f - v_i}{a}$

from the first equation of motion, we get

$2S = \frac{(v_f + v_i)(v_f - v_i)}{a}.$

Result

$2aS = v_f^2 - v_i^2$

SQ 2.4.10 How is the third equation of motion derived graphically?

Derivation

The area of the trapezium $OABD$ gives

$S = \frac{1}{2}(OA + BD) \times OD = \frac{1}{2}(v_i + v_f)t.$

Putting

$t = \frac{v_f - v_i}{a}$

gives

$2aS = v_f^2 - v_i^2$

SQ 2.4.11 When can vector quantities be manipulated like scalars in the equations of motion?

Condition

If the direction of motion does not change, then all the vector quantities can be manipulated like scalars.

Convention

The initial velocity is taken as positive, and a negative sign is assigned to quantities whose direction is opposite to that of the initial velocity.

SQ 2.4.12 A car travelling at 10 m s⁻¹ accelerates uniformly at 2 m s⁻². Find its velocity after 5 s.

Solution

Using the first equation of motion

$v_f = v_i + at$

Calculation

$v_f = 10 + (2)(5)$

Result

$v_f = 20\,\mathrm{m\,s^{-1}}$

SQ 2.4.13 A car with initial velocity 15 m s⁻¹ accelerates at 2 m s⁻² for 4 s. Find its displacement.

Solution

Using the second equation of motion

$S = v_i t + \frac{1}{2}at^2$

Calculation

$S = (15)(4) + \frac{1}{2}(2)(4)^2$

Result

$S = 76\,\mathrm{m}$

SQ 2.4.14 **A car starts from rest and reaches 300 km h⁻¹ over 0.45 km. Find its constant acceleration.**

Data

Here

$v_i = 0$

$v_f = 83.33\,\mathrm{m\,s^{-1}}$

and

$S = 450\,\mathrm{m}$

Solution

Using

$v_f^2 - v_i^2 = 2aS$

Result

$a = 7.72\,\mathrm{m\,s^{-2}}$

TOPIC 5 Motion Under Gravity

SQ 2.5.1 **What is the most familiar example of uniformly accelerated rectilinear motion?**

Answer

A body falling freely under the action of gravity is the most familiar example of uniformly accelerated rectilinear motion.

SQ 2.5.2 **What did Galileo state about freely falling bodies?**

Statement

According to Galileo, all bodies fall freely in vacuum under the acceleration due to gravity, denoted by g.

Meaning

Different bodies, when allowed to fall from the same height, strike the ground with the same velocity.

SQ 2.5.3 **What is the experimental value of acceleration due to gravity?**

Value

$g = 9.8\,\mathrm{m\,s^{-2}}$

SQ 2.5.4 **What is the sign convention for g?**

Convention

The value of g is taken positive for a falling body, when the initial velocity is zero.
It is taken negative for a body projected vertically upward, when the initial velocity is not zero.

SQ 2.5.5 **Write the three equations of motion for a freely falling body.**

Formulas

$v_f = v_i + gt$

$h = v_i t + \frac{1}{2}gt^2$

$v_f^2 - v_i^2 = 2gh$

SQ 2.5.6 **A ball is dropped from a tower and reaches the ground in 3.34 s. Find its velocity on striking the ground.**

Solution

Using

$v_f = v_i + gt$

with

$v_i = 0$

Calculation

$v_f = 0 + (9.8)(3.34)$

Result

$v_f = 32.7\,\mathrm{m\,s^{-1}}$

SQ 2.5.7 **A ball dropped from a tower strikes the ground at 32.7 m s⁻¹. Find the height of the tower.**

Solution

Using

$v_f^2 - v_i^2 = 2gh$

Calculation

$h = \frac{(32.7)^2}{2 \times 9.8}$

Result

$h = 54.56\,\mathrm{m}$

TOPIC 6 Projectile Motion

SQ 2.6.1 **What is observed when a ball is thrown horizontally from a certain height?**

Observation

The ball travels forward as well as falls downward, until it strikes something such as the ground.

SQ 2.6.2 **Why does the horizontal velocity of a projectile remain unchanged?**

Reason

According to Newton's first law of motion there will be no acceleration in the horizontal direction unless a horizontally directed force acts on the ball. Ignoring air friction, the only force acting during flight is the force of gravity, so there is no horizontal force.

SQ 2.6.3 **Write the expression for the horizontal distance covered by a projectile thrown horizontally.**

Formula

$x = v_i t$

The ball moves with a constant horizontal velocity component.

SQ 2.6.4 **Write the expression for the vertical distance covered by a projectile thrown horizontally.**

Explanation

The vertical motion is the same as for a freely falling body, and the initial vertical velocity is zero.

Formula

$y = \frac{1}{2}gt^2$

SQ 2.6.5 **Define projectile motion.**

Definition

Projectile motion is two dimensional motion under constant acceleration due to gravity.

SQ 2.6.6 Give three examples of projectiles.

Examples

A football kicked off by a player, a ball thrown by a cricketer, and a missile fired from a launching pad, all projected at some angle with the horizontal.

SQ 2.6.7 How is the motion of a projectile studied easily?

Method

The motion of a projectile can be studied easily by resolving it into horizontal and vertical components, which are independent of each other.

SQ 2.6.8 What are the horizontal and vertical accelerations of a projectile?

Horizontal Acceleration

$a_x = 0$

because air resistance is neglected and no other force acts along this direction.

Vertical Acceleration

$a_y = g$

directed downward.

SQ 2.6.9 Write the horizontal component of velocity of a projectile at any time.

Formula

$v_{fx} = v_{ix} = v_i \cos\theta$

The horizontal component remains constant.

SQ 2.6.10 Write the vertical component of velocity of a projectile at any time.

Formula

$v_{fy} = v_i \sin\theta - gt$

SQ 2.6.11 Write the magnitude of the velocity of a projectile at any instant.

Formula

$v = \sqrt{v_{fx}^2 + v_{fy}^2}$

SQ 2.6.12 How is the direction of the resultant velocity of a projectile found?

Formula

$\tan\phi = \dfrac{v_{fy}}{v_{fx}}$

SQ 2.6.13 Derive the expression for the height of a projectile.

Derivation

Using

$2aS = v_f^2 - v_i^2$

with

$a = -g$

initial vertical velocity $v_i \sin\theta$ and final vertical velocity zero at the highest point.

This gives

$-2gh = 0 - v_i^2 \sin^2\theta$

Result

$h = \dfrac{v_i^2 \sin^2\theta}{2g}$

SQ 2.6.14 What is the effect of air resistance on the height of a projectile?

Effect

In the presence of air resistance the upward velocity of the projectile decreases, and hence its height will also decrease.

SQ 2.6.15 Define the time of flight of a projectile.

Definition

The time taken by a body to cover the distance from the place of its projection to the place where it hits the ground is called the time of flight.

SQ 2.6.16 Derive the expression for the time of flight of a projectile.

Derivation

Taking

$S = 0$

because the body goes up and comes back to the same level, the equation

$S = v_i t - \dfrac{1}{2}gt^2$

becomes

$0 = v_i \sin\theta\, t - \dfrac{1}{2}gt^2$

Result

$t = \dfrac{2v_i \sin\theta}{g}$

SQ 2.6.17 Define the range of a projectile.

Definition

The maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

SQ 2.6.18 Derive the expression for the range of a projectile.

Derivation

The range is the horizontal component of the velocity of projection multiplied by the total time of flight, so

$R = v_i \cos\theta \times \dfrac{2v_i \sin\theta}{g}.$

Since

$2\sin\theta\cos\theta = \sin 2\theta$

Result

$R = \dfrac{v_i^2}{g}\sin 2\theta$

SQ 2.6.19 On what does the range of a projectile depend?

Answer

The range of the projectile depends upon the velocity of projection and the angle of projection.

SQ 2.6.20 At what angle is the range of a projectile maximum?

Derivation

For maximum range the factor $\sin 2\theta$ must be 1, so

$2\theta = 90^\circ$

Result

$\theta = 45^\circ$

SQ 2.6.21 **What is the effect of air resistance on the range of a projectile?**

Effect

Air resistance slows down the forward motion of the projectile, reducing its velocity v_i. The reduction in v_i results in a decrease in the range.

SQ 2.6.22 **Why is the actual trajectory of a projectile not perfectly parabolic?**

Reason

Air resistance is not constant throughout the flight. As the object slows down the air resistance also decreases, so the object retards and accelerates more slowly.

Result

This gives a trajectory that is skewed, with a steeper descent than ascent.

SQ 2.6.23 **What happens to the height and range when the angle of projection is larger than 45°?**

Answer

When the angle of projection is larger than 45°, the height attained will be more but the range will be less.

SQ 2.6.24 **A ball is thrown at 30 m s⁻¹ at 30° above the horizontal. Find its time of flight.**

Solution

Using

$$t = \frac{2v_i \sin\theta}{g}$$

Calculation

$$t = \frac{2 \times 30 \times 0.5}{9.8}$$

Result

$t = 3.1\,\mathrm{s}$

SQ 2.6.25 **A ball is thrown at 30 m s⁻¹ at 30° above the horizontal. Find the height to which it rises.**

Solution

Using

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

Calculation

$$h = \frac{(30)^2(0.5)^2}{19.6}$$

Result

$h = 11.4\,\mathrm{m}$

SQ 2.6.26 **A ball is thrown at 30 m s⁻¹ at 30° above the horizontal. Find its horizontal range.**

Solution

Using

$$R = \frac{v_i^2}{g}\sin 2\theta$$

Calculation

$$R = \frac{(30)^2 \times 0.866}{9.8}$$

Result

$R = 79.53\,\mathrm{m}$

SQ 2.6.27 **At what angle does a projectile gain its maximum height?**

Answer

Since

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

is maximum when

$\sin\theta = 1$

the projectile gains its maximum height at

$\theta = 90^\circ$

SQ 2.6.28 **For which two angles is the range of a projectile the same?**

Answer

The range is the same for two angles which are mutually complementary, because $\sin 2\theta$ has the same value for θ and $90^\circ - \theta$.

SQ 2.6.29 **What is the acceleration at the top of the trajectory of a projectile?**

Answer

The acceleration at the top of the trajectory is g, directed vertically downward, because gravity acts throughout the flight.

TOPIC 7 Momentum

SQ 2.7.1 **What property of a moving object did Newton refer to as momentum?**

Answer

A moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. Newton referred to this property as momentum.

SQ 2.7.2 **Why is a faster or more massive object harder to stop?**

Reason

The faster an object is travelling, the harder it is to stop it. Similarly, if two objects move with the same velocity, it is more difficult to stop the more massive of the two.

SQ 2.7.3 **Define linear momentum and write its formula.**

Definition

Linear momentum is a vector quantity defined as the product of an object's mass and velocity.

Formula

$\mathbf{p} = m\mathbf{v}$

SQ 2.7.4 **What is the direction of linear momentum?**

Answer

Linear momentum is a vector quantity and has the direction of the velocity.

SQ 2.7.5 **Write the SI unit of momentum.**

SI Unit

The SI unit of momentum is kilogram metre per second ($\mathrm{kg\,m\,s^{-1}}$). It can also be expressed as newton second ($\mathrm{N\,s}$).

SQ 2.7.6 **Show that the rate of change of momentum is equal to the applied force.**

Derivation

The acceleration produced by a force is

$a = \dfrac{v_f - v_i}{t}$

and by Newton's second law

$a = \dfrac{F}{m}$

Equating the two gives

$F \times t = mv_f - mv_i.$

Result

$F = \dfrac{mv_f - mv_i}{t}$

SQ 2.7.7 **State Newton's second law of motion in terms of momentum.**

Statement

The time rate of change of momentum of a body is equal to the applied force.

SQ 2.7.8 **Why is the momentum form of Newton's second law more general than F = ma?**

Reason

It can easily be extended to account for changes as the body accelerates when its mass also changes.

Example

As a rocket accelerates it loses mass, because its fuel is burnt and ejected to provide greater thrust.

SQ 2.7.9 **When is it more convenient to use the product of force and time?**

Answer

When the applied force is not constant and acts for a very short time, as when a bat hits a cricket ball, it is more convenient to deal with the product $F \times t$ instead of either quantity alone.

SQ 2.7.10 **Define impulse and relate it to momentum.**

Definition

The product of the average force F that acts during time t is called impulse.

Formula

$\text{Impulse} = F \times t = mv_f - mv_i$

Thus impulse is equal to the change in momentum.

SQ 2.7.11 **A 1500 kg car has its velocity reduced from 20 m s^{-1} to 15 m s^{-1} in 3.0 s. Find the average retarding force.**

Solution

Using

$F \times t = mv_f - mv_i$

Calculation

$F \times 3.0 = 1500(15) - 1500(20)$

Result

$F = -2.5\,\text{kN}$

The negative sign indicates that the force is a retarding one.

SQ 2.7.12 **What is an isolated system?**

Definition

An isolated system is a system on which no external agency exerts any force.

Example

The molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system.

SQ 2.7.13 **State the law of conservation of momentum.**

Statement

The total linear momentum of an isolated system remains constant.

SQ 2.7.14 **Under what condition does the law of conservation of momentum hold?**

Condition

It holds for an isolated system, that is a system on which no external agency exerts any force.

SQ 2.7.15 **Derive the law of conservation of momentum for two colliding balls.**

Derivation

For the two balls,

$F \times t = m_1 v_1' - m_1 v_1$

and

$F' \times t = m_2 v_2' - m_2 v_2$

Since the action force is equal and opposite to the reaction force,

$F + F' = 0$

so adding the two expressions gives zero on the left hand side.

Result

$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

SQ 2.7.16 **What must be noticed while applying the law of conservation of momentum?**

Answer

We must notice that the momentum of a body is a vector quantity, so its direction must be taken into account.

SQ 2.7.17 **Two balls of 2.0 kg and 3.0 kg move towards each other at 6.0 m s^{-1} and 4 m s^{-1}. Find the momentum of the system before collision.**

Solution

Since the balls move towards one another, their velocities are of opposite sign.

Calculation

$p = 2(6.0) + 3(-4)$

Result

$p = 0$

SQ 2.7.18 **For the above two balls, find the velocity of the smaller ball after collision if the bigger ball moves at 3.0 m s^{-1}.**

Solution

Since the total momentum before collision is zero, the total momentum after collision must also be zero, so

$0 = 2v_1' + 3(-3).$

Result

$v_1' = 4.5\,\text{m}\,\text{s}^{-1}$

TOPIC 8 Elastic and Inelastic Collisions

SQ 2.8.1 Why does a tennis ball dropped on the floor not rebound to its initial height?

Reason

A portion of the kinetic energy is lost, partly due to friction as the molecules in the ball move past one another when the ball distorts, and partly due to its change into heat and sound energies.

SQ 2.8.2 Define an inelastic collision.

Definition

A collision in which the kinetic energy of the system is not conserved is called an inelastic collision.

SQ 2.8.3 Define an elastic collision.

Definition

A collision in which no kinetic energy is lost is said to be an elastic collision.

Example

When a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height and loses a negligible amount of energy.

SQ 2.8.4 Which quantities are conserved in all types of collisions?

Answer

Momentum and total energy are conserved in all types of collisions. However, the kinetic energy is conserved only if the collision is elastic.

SQ 2.8.5 Differentiate between elastic and inelastic collisions.

Elastic Collision	Inelastic Collision
The kinetic energy of the system is conserved.	The kinetic energy of the system is not conserved.
No kinetic energy is lost during the collision.	Part of the kinetic energy changes into heat and sound.
A hard ball rebounding from a marble floor is an example.	A tennis ball that does not rebound to its initial height is an example.

SQ 2.8.6 Write the momentum equation for an elastic collision in one dimension.

Formula

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$
$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

SQ 2.8.7 Write the kinetic energy equation for an elastic collision in one dimension.

Formula

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

SQ 2.8.8 Show that the relative velocity of approach equals the relative velocity of separation.

Derivation

Dividing the kinetic energy equation by the momentum equation gives
$v_1 + v_1' = v_2 + v_2'$.

Result

$v_1 - v_2 = -(v_1' - v_2')$

Thus the relative velocities before and after the collision have the same magnitude but are reversed.

SQ 2.8.9 Write the final velocity of the first body after a one dimensional elastic collision.

Formula

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$$

SQ 2.8.10 Write the final velocity of the second body after a one dimensional elastic collision.

Formula

$$v_2' = \frac{m_2 - m_1}{m_1 + m_2}v_2 + \frac{2m_1}{m_1 + m_2}v_1$$

SQ 2.8.11 What happens in an elastic collision when the two masses are equal?

Answer

When
$m_1 = m_2$
the bodies simply exchange their velocities, so
$v_1' = v_2$
and
$v_2' = v_1$

SQ 2.8.12 What happens when a body collides elastically with an equal mass at rest?

Answer

When
$m_1 = m_2$
and
$v_2 = 0$
then
$v_1' = 0$
and
$v_2' = v_1$

Example

When a billiard ball collides with an exactly similar ball at rest, the first ball stops while the second begins to move with the velocity the first had.

SQ 2.8.13 What happens when a light body collides elastically with a massive body at rest?

Answer

Here
$v_2 = 0$
and $m_2 \gg m_1$, so
$v_1' = -v_1$
and
$v_2' = 0$

Meaning

The light body bounces back with the same velocity while the massive body remains stationary. This fact is used by the squash player.

SQ 2.8.14 **What happens when a massive body collides elastically with a light stationary body?**

Answer
Here $m_1 \gg m_2$ and
$v_2 = 0$
so
$v_1' = v_1$
and
$v_2' = 2v_1$

Meaning
There is practically no change in the velocity of the massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body.

SQ 2.8.15 **A 70 g ball moving at 9 m s⁻¹ hits a stationary 140 g ball elastically. Find the velocity of the first ball after collision.**

Solution
Using
$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

Calculation
$$v_1' = \frac{70 - 140}{70 + 140} \times 9$$

Result
$$v_1' = -3\,\mathrm{m\,s^{-1}}$$

SQ 2.8.16 **A 70 g ball moving at 9 m s⁻¹ hits a stationary 140 g ball elastically. Find the velocity of the second ball after collision.**

Solution
Using
$$v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

Calculation
$$v_2' = \frac{2 \times 70}{70 + 140} \times 9$$

Result
$$v_2' = 6\,\mathrm{m\,s^{-1}}$$

TOPIC 9 Inelastic Collision in One Dimension

SQ 2.9.1 **What is a perfectly inelastic collision in one dimension?**

Definition
It is a collision in which two bodies moving along the same line collide and stick together, so that their combined mass moves with a single final velocity.

SQ 2.9.2 **Which body is regarded as the projectile and which as the target?**

Answer
When two bodies move along the same line with $v_1 > v_2$, then m_1 is regarded as the projectile and m_2 as the target.

SQ 2.9.3 **Derive the common velocity after a one dimensional inelastic collision.**

Derivation
Since the total momentum is conserved,
$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f.$

Result
$$v_f = \frac{m_1}{m_1 + m_2} v_1 + \frac{m_2}{m_1 + m_2} v_2$$

SQ 2.9.4 **Write the common velocity when the target is at rest in an inelastic collision.**

Formula
$$v_f = \frac{m_1}{m_1 + m_2} v_1$$
It shows that the velocity of m_1 is reduced by the mass ratio $\dfrac{m_1}{m_1 + m_2}$.

SQ 2.9.5 **Which quantity is conserved in a perfectly inelastic collision?**

Answer
The total momentum of the bodies is conserved, but the kinetic energy is not conserved.

TOPIC 10 Elastic Collision in Two Dimensions

SQ 2.10.1 **Which two laws are applied to an elastic collision in two dimensions?**

Answer
Since the collision is elastic, both the law of conservation of momentum and the law of conservation of kinetic energy are applied.

SQ 2.10.2 **Why is momentum resolved into components in a two dimensional collision?**

Reason
Momentum is a vector quantity, so it is resolved into its rectangular components and the law of conservation of momentum is applied along both axes separately.

SQ 2.10.3 **Write the momentum conservation equation along the x-axis for a two dimensional elastic collision.**

Formula
$$m_1 v_1 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2$$

SQ 2.10.4 **Write the momentum conservation equation along the y-axis for a two dimensional elastic collision.**

Formula
$$0 = m_1 v_1' \sin\theta_1 - m_2 v_2' \sin\theta_2$$

SQ 2.10.5 **Write the energy conservation equation for a two dimensional elastic collision.**

Formula
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

TOPIC 11 Inelastic Collision in Two Dimensions

SQ 2.11.1 **What is a perfect inelastic collision in two dimensions?**

Definition
The perfect inelastic collision is one in which the colliding objects stick together to make a single mass after collision.

SQ 2.11.2 Are macroscopic collisions generally elastic or inelastic?

Answer
The macroscopic collisions are generally inelastic and do not conserve kinetic energy.

SQ 2.11.3 Write the momentum equation in the x-direction for a two dimensional inelastic collision.

Formula
$m_1v_1 + m_2v_2\cos\theta = Mv_f\cos\phi$
Here
$M = m_1 + m_2$
is the combined mass.

SQ 2.11.4 Write the momentum equation in the y-direction for a two dimensional inelastic collision.

Formula
$m_2v_2\sin\theta = Mv_f\sin\phi$

SQ 2.11.5 Write the initial kinetic energy of a system before a two dimensional collision.

Formula
$(KE)_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
Since kinetic energy is a scalar quantity, the velocities need not be broken into components.

SQ 2.11.6 Write the final kinetic energy after a perfectly inelastic two dimensional collision.

Formula
$(KE)_f = \frac{1}{2}Mv_f^2$
Here v_f is the magnitude of the final velocity of the combined mass.

SQ 2.11.7 How is the energy loss in an inelastic collision computed?

Formula
$\Delta KE = (KE)_i - (KE)_f$
This lost kinetic energy is transformed into other forms of energy such as heat, sound or deformation.

SQ 2.11.8 Explain a karate chop breaking bricks as an inelastic collision.

Explanation
The objects involved do not bounce back after impact. Some of the energy from the strike is absorbed by the bricks, converting into heat, sound and the force needed to break them.
Result
The energy goes into breaking the bricks rather than causing the hand to rebound.

SQ 2.11.9 Explain a car crash as an inelastic collision.

Explanation
When the vehicles collide they absorb the impact energy, causing them to crumple and deform. This energy absorption slows the cars down and stops them from bouncing back.
Result
Most of the kinetic energy is lost, turning into heat, sound and damage to the vehicles.

SQ 2.11.10 Explain why a bat and ball collision is inelastic.

Explanation
When the bat hits the ball, some of the kinetic energy is lost because the ball deforms, and energy is also converted into heat and sound.
Reason
Even though the bat is rigid it does not transfer energy perfectly and absorbs some energy itself, so not all the initial kinetic energy is conserved.

TOPIC 12 Rocket Propulsion

SQ 2.12.1 How does a rocket move?

Answer
Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines.

SQ 2.12.2 On which principle does rocket propulsion work?

Principle
The rocket gains momentum equal to the momentum of the gas expelled from the engine, but in the opposite direction.

SQ 2.12.3 Why does a rocket get faster and faster?

Reason
The rocket engines continue to expel gases after the rocket has begun moving, so the rocket continues to gain more and more momentum. Instead of travelling at a steady speed it gets faster and faster so long as the engines are operating.

SQ 2.12.4 Why can a rocket work at great heights?

Reason
A rocket carries its own fuel in the form of liquid or solid hydrogen and oxygen. It can therefore work at great heights where very little or no air is present.

SQ 2.12.5 How much fuel does a typical rocket consume?

Answer
In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about $10000\,\mathrm{kg\,s^{-1}}$ of fuel and ejects the burnt gases at speeds of over $4000\,\mathrm{m\,s^{-1}}$.

SQ 2.12.6 What fraction of the launch mass of a rocket is fuel?

Answer
More than 80% of the launch mass of a rocket consists of fuel only.

SQ 2.12.7 **How is the problem of the mass of fuel overcome?**

Answer
One way is to make the rocket from several rockets linked together. When one rocket has done its job it is discarded, leaving the others to carry the spacecraft further up at ever greater speed.

SQ 2.12.8 **Write the expression for the acceleration of a rocket.**

Derivation
If m is the mass of gases ejected per second with velocity v relative to the rocket, the change in momentum per second is mv, which equals the thrust produced.

Formula
$$a = \frac{mv}{M}$$

Here M is the mass of the rocket.

SQ 2.12.9 **Why does the acceleration of a rocket increase as it moves upward?**

Reason
When the fuel in the rocket is burned and ejected, the mass M of the rocket decreases. Since

$$a = \frac{mv}{M}$$

the acceleration therefore increases.

SQ 2.12.10 **What happens in the combustion chamber of a rocket?**

Answer
Fuel and oxygen mix in the combustion chamber. The hot gases exhaust the chamber at a very high velocity, and the gain in momentum of the gases equals the gain in momentum of the rocket.