

Topic 4.1 Work Done by a Constant Force

LQ 2 starts here — How is work done by a (i) constant force (ii) variable force?

Work in Physics

In everyday speech work is thought of as physical or mental effort. In physics the term involves two things, a force and a displacement, and no work is done unless both are present.

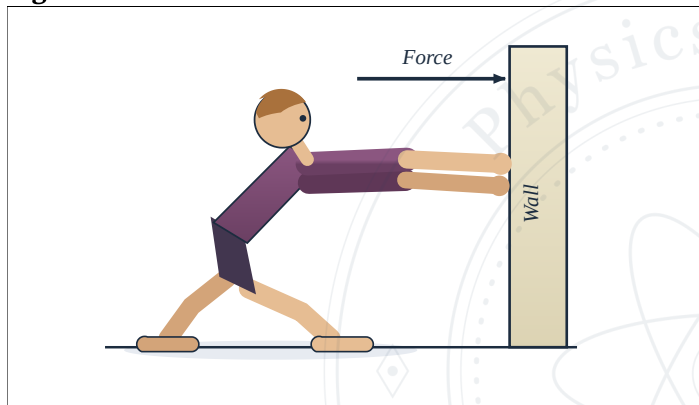
Work Done by a Constant Force

Consider an object being pulled by a constant force F which displaces it through a displacement d in the direction of the force. The work W is defined as the product of the magnitude of the force and the magnitude of the displacement.

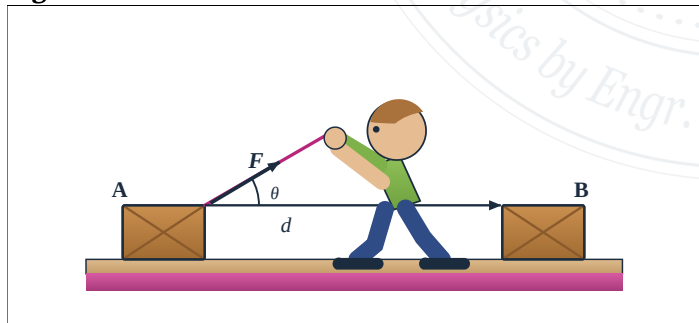
Mathematically $W = Fd$

Zero Displacement

If the displacement is zero, no work is done however large the force applied. Pushing on a wall may tire the muscles, but the work done is zero.

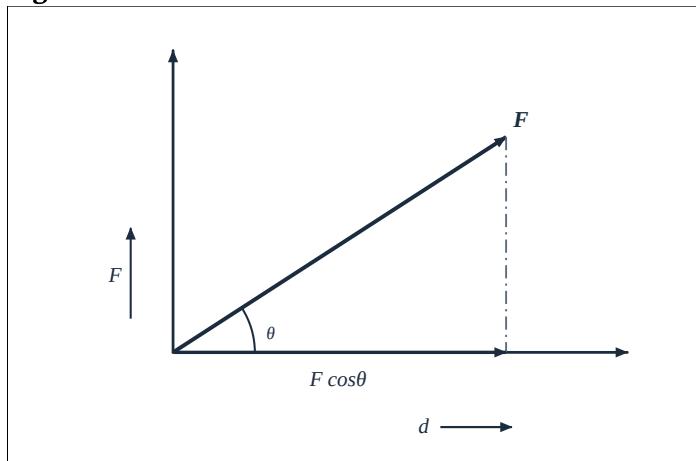
Figure

Force at an Angle to the Displacement

The force applied on a body is not always along the displacement. If the force F makes an angle θ with the displacement d , the work done is the product of the component of the force along the displacement and the magnitude of the displacement.

Figure

Component Along the Displacement

The component of the force along the displacement is $F \cos \theta$, and the work done is that component multiplied by the displacement.

$$W = (F \cos \theta) d = Fd \cos \theta$$

Figure

General Form

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Scalar Quantity

The work done is the scalar product of the force and the displacement, so work itself is a scalar quantity. It has magnitude only and no direction.

Unit of Work

The SI unit of work is the newton metre, which is given the name joule.

$$W = Fd$$

$$1 \text{ J} = 1 \text{ N} \times 1 \text{ m} = 1 \text{ N m}$$

Definition of a Joule

One joule is the work done when a force of one newton displaces a body through one metre in the direction of the force.

Sign of the Work Done

The angle between the force and the displacement decides the sign of the work.

- If the angle is less than 90° , the work done is positive.
- If the angle is a right angle, no work is done, because the cosine of a right angle is zero.
- If the angle is greater than 90° , the work done is negative.

Work from a Force-Displacement Graph

When a constant force acts through a distance d , the event can be plotted on a graph with the distance along the x-axis and the force along the y-axis.

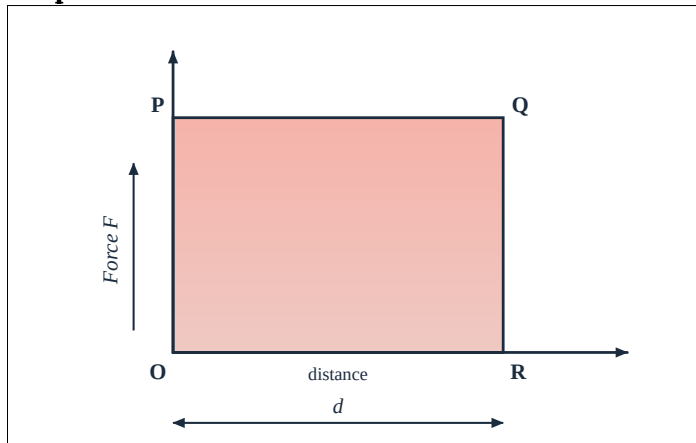
Shape of the Graph

The force does not vary, so the graph is a horizontal straight line.

Area Under the Graph

If the constant force F and the displacement d are in the same direction, the work done is Fd , and the shaded rectangle under the line has exactly that area.

$$W = \text{area under the graph}$$

Graph


Conclusion

The area under a force-displacement curve represents the work done by the force. When the force is not along the displacement, the graph is plotted between $F \cos \theta$ and d instead.

Short Questions

SQ 4.1.1 A man pushes hard against a wall but the wall does not move. Is any work done? Explain.

No Work is Done

Work in physics needs a force and a displacement in the direction of that force.

Reason

The wall does not move, so the displacement is zero and the work done is zero however large the force applied.

$$W = Fd = F \times 0 = 0$$

Effort

The muscles do tire, but that is physiological effort and not work in the sense of physics.

SQ 4.1.2 Under what condition is the work done by a force zero even though both the force and the displacement are not zero?

Force Perpendicular to the Displacement

The work is zero when the force is perpendicular to the displacement.

Reason

The work depends on the component of the force along the displacement, and the cosine of a right angle is zero.

$$W = Fd \cos 90^\circ$$

$$W = 0$$

Example

A coolie carrying a load on his head walking on level ground does no work against the weight of the load.

SQ 4.1.3 Show that work is a scalar quantity although force and displacement are both vectors.

Scalar Product

The work done is defined as the scalar product of the force and the displacement.

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Reason

The scalar product of two vectors is a scalar, because the result carries only a magnitude and the direction information is used up in the angle.

Result

Work therefore has magnitude only and no direction.

SQ 4.1.4 A force of 10 N moves a body through 5 m, acting at 60° to the displacement. Find the work done.

Given Data $F = 10 \text{ N}$

$$d = 5 \text{ m}$$

$$\theta = 60^\circ$$

Formula $W = Fd \cos \theta$

Solution

$$W = 10 \times 5 \times \cos 60^\circ = 50 \times 0.5 = 25 \text{ J}$$

Result $W = 25 \text{ J}$

SQ 4.1.5 When is the work done by a force said to be negative? Give one example.

Negative Work

The work is negative when the angle between the force and the displacement is greater than 90° , because the cosine of such an angle is negative.

$$W = Fd \cos \theta$$

Example

When a body is lifted upward, the work done by its weight is negative, since the weight acts downward

while the displacement is upward.

SQ 4.1.6 How is the work done by a constant force found from a force-displacement graph?

Work from the Graph

The distance is plotted along the x-axis and the force along the y-axis. A constant force gives a horizontal straight line.

Area

The rectangle under that line has a height F and a base d , so its area is the product of the two.

$$W = Fd = \text{area under the graph}$$

When the Force is at an Angle

The graph is then plotted between $F \cos \theta$ and d instead.

MCQ 4.1.1 Work done by a constant force is the:

A vector product of force and displacement	B scalar product of force and displacement
C ratio of force to displacement	D sum of force and displacement

Solution: Work is defined as $\vec{F} \cdot \vec{d}$, and the scalar product of two vectors is a scalar.

MCQ 4.1.2 If the angle between the force and the displacement is 90° , the work done is:

A maximum	B negative
C zero	D equal to Fd

Solution: $W = Fd \cos 90^\circ$ and the cosine of a right angle is zero.

MCQ 4.1.3 The SI unit of work is equivalent to:

A newton metre	B newton per metre
C newton second	D newton per second

Solution: $W = Fd$, so the unit of work is N m, which is given the name joule.

MCQ 4.1.4 A force of 5 N moves a body through 4 m at 60° to the displacement. The work done is:

A 20 J	B 10 J
C 17.3 J	D zero

Solution: $W = 5 \times 4 \times \cos 60^\circ = 20 \times 0.5 = 10 \text{ J}$

MCQ 4.1.5 However large the applied force, no work is done if the:

A force is small	B body is heavy
C displacement is zero	D surface is rough

Solution: $W = Fd$, so a zero displacement makes the work zero whatever the force.

MCQ 4.1.6 The work done by a force is negative when the angle between the force and the displacement is:

A zero	B less than 90°
C equal to 90°	D greater than 90°

Solution: The cosine of an angle greater than a right angle is negative, so $Fd \cos \theta$ is negative.

MCQ 4.1.7 The area under a force-displacement graph gives the:

A power	B work done
C momentum	D acceleration

Solution: For a constant force the area of the rectangle under the line is Fd , which is the work done.

Topic 4.2 Work Done by a Variable Force

Need for a New Method

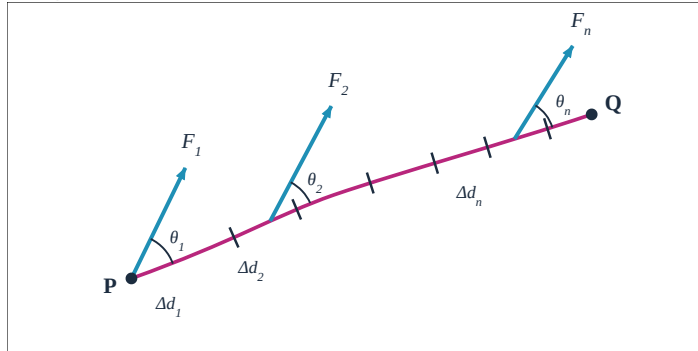
In many cases the force does not remain constant while the work is being done. As a rocket moves away from the Earth the work is done against gravity, which varies as the inverse square of the distance from the centre of the Earth, and the force exerted by a spring increases with the amount of stretch.

Dividing the Path

Consider a particle moving in the x-y plane from a point P to a point Q . The path is divided into n short intervals of displacement $\Delta d_1, \Delta d_2, \dots, \Delta d_n$, and

$F_1, F_2, \dots, F_n$ are the forces acting during these intervals.

Diagram



Work in One Interval

During each small interval the force is supposed to be approximately constant, so the work done in the first interval is the ordinary constant-force expression.

$$\Delta W_1 = \vec{F}_1 \cdot \Delta \vec{d}_1 = F_1 \cos \theta_1 \Delta d_1$$

Second Interval

$$\Delta W_2 = \vec{F}_2 \cdot \Delta \vec{d}_2 = F_2 \cos \theta_2 \Delta d_2$$

Total Work Done

The total work done in moving the object from P to Q is found by adding all such terms.

$$W_{PQ} = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n$$

$$W_{PQ} = F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + \dots + F_n \cos \theta_n \Delta d_n$$

$$W_{PQ} = \sum_{i=1}^n F_i \cos \theta_i \Delta d_i$$

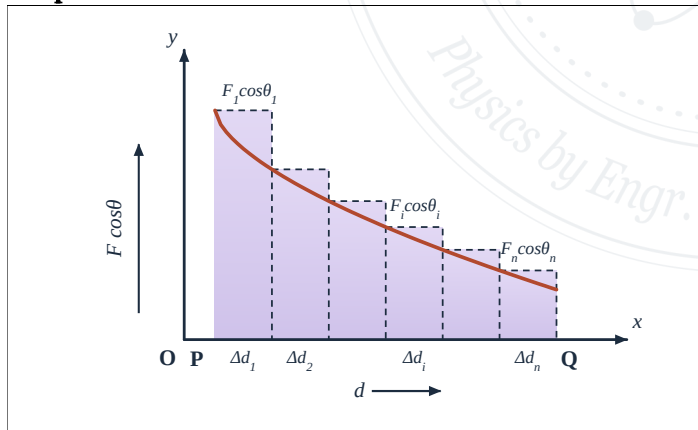
Graphical Treatment

The same result is examined graphically by plotting $F \cos \theta$ against d . The displacement is subdivided into the same n equal intervals, and the value of $F \cos \theta$ at the beginning of each interval is marked.

Area of One Rectangle

The i^{th} shaded rectangle has an area $F_i \cos \theta_i \Delta d_i$, which is the work done during the i^{th} interval. The total work is therefore the sum of the areas of all the rectangles.

Graph



Making the Result Exact

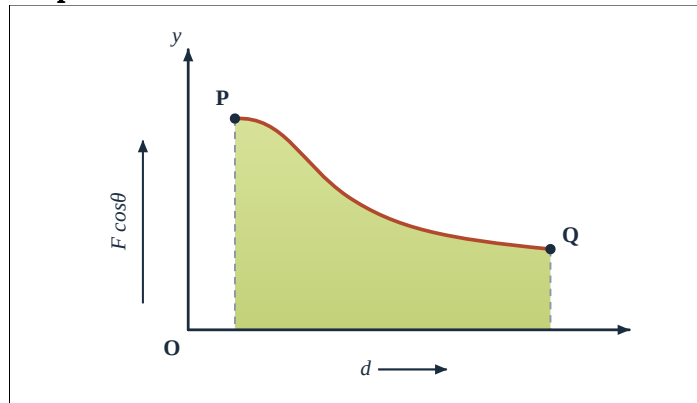
The smaller each interval is made, the more accurate the sum becomes. Letting each interval approach zero gives an exact result for the work done.

$$W = \lim_{\Delta d \rightarrow 0} \sum_{i=1}^n F_i \cos \theta_i \Delta d_i$$

Meaning of the Limit

As the intervals approach zero, the total area of all the rectangles approaches the area between the $F \cos \theta$ curve and the x-axis from P to Q.

Graph



Conclusion

The work done by a variable force in moving a particle between two points is equal to the area under the $F \cos \theta$ against d curve between those two points.

LQ 2 ends here

Book Examples

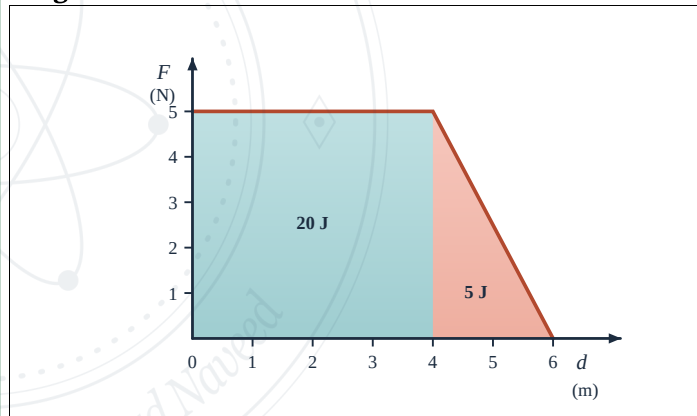
Example 4.2.1 A force F acting on an object varies with the distance d as shown in the graph. The force is constant at 5 N from $d = 0$ to $d = 4$ m and then falls uniformly to zero at $d = 6$ m. Calculate the work done by the force as the object moves from $d = 0$ to $d = 6$ m.

Given Data

$$F = 5 \text{ N } (0 \leq d \leq 4 \text{ m})$$

$$d = 0 \text{ to } 6 \text{ m}$$

Diagram



To Find

Work done by the force $W = ?$

Formula

The work done by a variable force is the area under the force-displacement curve.

$$W = \text{area under the curve}$$

Solution

The area under the curve splits into a rectangle over the first four metres and a triangle over the last two metres.

$$\text{Area of the Rectangle } W_1 = 4 \text{ m} \times 5 \text{ N} = 20 \text{ J}$$

$$\text{Area of the Triangle } W_2 = \frac{1}{2} \times 2 \text{ m} \times 5 \text{ N} = 5 \text{ J}$$

$$\text{Total Work Done } W = 20 \text{ J} + 5 \text{ J}$$

$$\text{Result } W = 25 \text{ J}$$

Short Questions

SQ 4.2.1 Why can the relation $W = Fd \cos \theta$ not be used directly when the force is variable?

Force Not Constant

The relation assumes that the force keeps the same magnitude and direction throughout the displacement.

Variable Force

A variable force has a different value at every point of the path, so there is no single F to put into the relation.

Way Out

The path is broken into intervals so short that the force may be taken as constant over each one, and the work done in all of them is added.

SQ 4.2.2 How is the work done by a variable force calculated by dividing the path?

Dividing the Path

The path from P to Q is divided into n short intervals $\Delta d_1, \Delta d_2, \dots, \Delta d_n$, over each of which the force is nearly constant.

Work in One Interval $\Delta W_i = F_i \cos \theta_i \Delta d_i$

Total Work

Adding the work done in all the intervals gives the total work.

$$W = \sum_{i=1}^n F_i \cos \theta_i \Delta d_i$$

SQ 4.2.3 What does the area under the $F \cos \theta$ against d curve represent?

Area Under the Curve

It represents the work done by the variable force in moving the particle between the two points.

Reason

Each narrow strip of the graph has an area $F_i \cos \theta_i \Delta d_i$, which is exactly the work done over that interval, and the whole area is the sum of all such strips.

SQ 4.2.4 Why does making the intervals smaller give a more accurate value of the work done?

Shorter Interval, Better Fit

The force is treated as constant over each interval, and that assumption is better the shorter the interval is.

Limit

As each interval approaches zero the rectangles fit the curve exactly, so the sum becomes an exact result.

$$W = \lim_{\Delta d \rightarrow 0} \sum F_i \cos \theta_i \Delta d_i$$

SQ 4.2.5 Give two examples of a variable force.

Gravitational Force on a Rocket

As a rocket moves away from the Earth, the force of gravity on it varies as the inverse square of the distance from the centre of the Earth.

Spring Force

The force exerted by a spring increases with the amount by which the spring is stretched or compressed.

MCQ 4.2.1 The work done by a variable force is equal to the area under the curve between:

- | | |
|---------------|---------------------------|
| A F and t | B $F \cos \theta$ and d |
| C v and t | D F and v |

Solution: The strips of the $F \cos \theta$ against d graph each give the work done over one interval, so the whole area is the work done.

MCQ 4.2.2 As a rocket moves away from the Earth, the force of gravity on it varies as:

- | | |
|---|------------------------------|
| A the distance | B the square of the distance |
| C the inverse square of the distance | D it stays constant |

Solution: The gravitational force follows an inverse square law in the distance from the centre of the Earth.

MCQ 4.2.3 In finding the work done by a variable force, the force is taken as constant over:

- | | |
|------------------------------|------------------------------|
| A the whole path | B each short interval |
| C the first half of the path | D no part of the path |

Solution: The path is divided into intervals short enough for the force to be treated as constant over each one.

MCQ 4.2.4 The force exerted by a spring is a variable force because it:

- | | |
|-------------------------------------|---|
| A acts only at the ends | B is always zero |
| C acts perpendicular to the stretch | D increases with the amount of stretch |

Solution: The spring force grows as the spring is stretched or compressed further, so it is not constant.

MCQ 4.2.5 The area of the i^{th} strip of the $F \cos \theta$ against d graph represents the:

- | | |
|-------------------------------------|--------------------------|
| A total work done | B power in that interval |
| C work done in that interval | D average force |

Solution: The strip has an area $F_i \cos \theta_i \Delta d_i$, which is exactly the work done over that interval.

MCQ 4.2.6 The result for the work done by a variable force becomes exact when each interval:

- | | |
|--------------------------|----------------------|
| A approaches zero | B becomes very large |
| C equals the whole path | D equals one metre |

Solution: In the limit as $\Delta d \rightarrow 0$ the rectangles fit the curve exactly, so the sum becomes the true area.

Topic 4.3 Conservative and Non-Conservative Forces

LQ 3 starts here — Define conservative field. Show that gravitational field is conservative in nature.

Conservative Field

A field in which the work done in moving a body between two points is independent of the path followed, and the work done around a closed path is zero, is called a conservative field.

Examples

- Gravitational field.
- Electric field.

Gravitational Field

The space around the Earth in which its gravitational force acts on a body is called the gravitational field.

W by Gravitational Field

When a body is moved in the gravitational field, work is done on it by the gravitational force, and that force is the weight of the body.

$$F = mg$$

$$W = Fd \cos \theta = mgd \cos \theta$$

Positive Work

The work is positive when the displacement is along the gravitational force, that is when the body moves downward.

$$\theta = 0^\circ$$

Negative Work

The work is negative when the displacement is against the gravitational force, that is when the body is raised upward.

$$\theta = 180^\circ$$

Zero Work

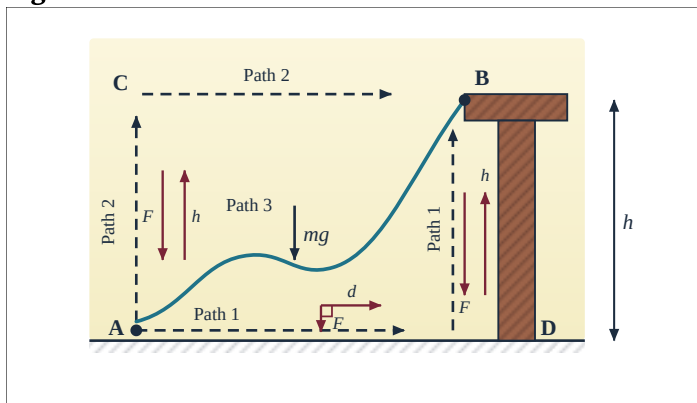
The work is zero when the displacement is perpendicular to the gravitational force, that is along a horizontal path.

$$\theta = 90^\circ$$

W along Different Paths

An object of mass m is displaced with a constant speed from a point A to a point B through a height h , along three different paths in the gravitational field.

Figure



Path-I (ADB)

$$W_{ADB} = W_{AD} + W_{DB}$$

$$W_{AD} = mg(AD) \cos 90^\circ = 0$$

$$W_{DB} = mgh \cos 180^\circ = -mgh$$

$$W_{ADB} = 0 + (-mgh)$$

$$W_{ADB} = -mgh$$

Path-II (ACB)

$$W_{ACB} = W_{AC} + W_{CB}$$

$$W_{AC} = mgh \cos 180^\circ = -mgh$$

$$W_{CB} = mg(CB) \cos 90^\circ = 0$$

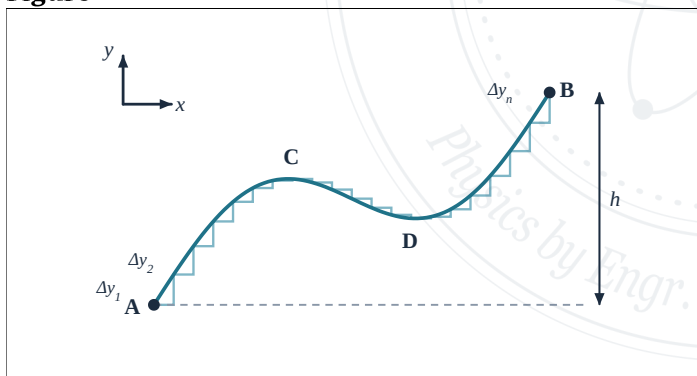
$$W_{ACB} = -mgh + 0$$

$$W_{ACB} = -mgh$$

Path-III (Curved Path)

- The curved path is broken into a series of small horizontal and vertical steps.
- No work is done along the horizontal steps, so only the vertical steps count, and they add up to the height h .

Figure



$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \dots + \Delta y_n)$$

$$\Delta y_1 + \Delta y_2 + \dots + \Delta y_n = h$$

$$W_{AB} = -mgh$$

Along Closed Path

$$W_{closed} = W_{ACB} + W_{BDA} = -mgh + mgh = 0$$

Result

The work done by the gravitational force is the same along all three paths and is zero around a closed path, so the gravitational field is a conservative field.

Other Conservative Forces

- The electrostatic force.
- The elastic spring force.

LQ 3 ends here

Non-Conservative Force

A force is non-conservative if the work done by it in moving an object between two points, or in a closed path, depends on the path of the motion.

Kinetic Friction

When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path.

$$W_f = -fd$$

Why Friction is Non-Conservative

A greater amount of work is done over a longer path between the same two points, so the work depends on the choice of path. The total work done by a non-conservative force in a closed path is also not zero.

Examples of Non-Conservative Forces

- Air resistance.
- Tension in a string.
- The normal force.
- The propulsion force of a rocket.

Short Questions

SQ 4.3.1 Show that the work done by gravity in moving a body from A to B is the same along all paths.

Path 1

The horizontal part does no work and the vertical part gives the whole of it.

$$W_{ADB} = 0 + (-mgh) = -mgh$$

Path 2

$$W_{ACB} = -mgh + 0 = -mgh$$

Path 3

A curved path is broken into horizontal and vertical steps, and only the vertical steps count.

$$W_{AB} = -mg(\Delta y_1 + \dots + \Delta y_n) = -mgh$$

Result

The work is the same along all three paths, so it is independent of the path.

SQ 4.3.2 Why is no work done along the horizontal steps of a curved path in a gravitational field?

Weight Perpendicular to the Step

Along a horizontal step the weight mg acts vertically downward while the displacement is horizontal, so the two are perpendicular.

$$W = mgd \cos 90^\circ$$

$$W = 0$$

Result

Work is done by gravity only during the vertical displacements of the path.

SQ 4.3.3 Why is the work done by a conservative force around a closed path zero?

Work Depends Only on the End Points

The work done by a conservative force depends only on the end points and not on the path taken.

Closed Path

In a closed path the starting point and the finishing point are the same, so the work done going out is exactly cancelled by the work done coming back.

$$W_{closed} = 0$$

SQ 4.3.4 Give three examples of non-conservative forces.

Non-Conservative Forces

- Air resistance.
- Tension in a string.
- The propulsion force of a rocket.

Common Property

For each of them the work done in moving a body between two points depends on the path followed, and the work done in a closed path is not zero.

SQ 4.3.5 Why does a longer path mean a greater amount of work done against friction?

Kinetic Friction

The frictional force always acts opposite to the motion, so it does negative work equal to the force multiplied

by the length of the path.

$$W_f = -fd$$

Result

A longer path between the same two points means a larger d and therefore more work, which is why friction is a non-conservative force.

SQ 4.3.6 What is meant by a gravitational field?

Gravitational Field

The space around the Earth in which its gravitational force acts on a body is called the gravitational field.

Work in the Field

When an object is moved in this field, work is done by the gravitational force. The work is positive if the displacement is along the force and negative if it is against the force.

MCQ 4.3.1 The work done by the gravitational force in moving a body between two points depends on:

- | | |
|----------------------------------|-------------------------|
| A the length of the path | B the shape of the path |
| C only the two end points | D the speed of the body |

Solution: Gravity is a conservative force, so its work is independent of the path followed.

MCQ 4.3.2 The work done by a conservative force in a closed path is:

- | | |
|------------|------------------|
| A maximum | B zero |
| C negative | D equal to mgh |

Solution: The work depends only on the end points, and in a closed path they are the same point.

MCQ 4.3.3 Which one of the following is NOT a conservative force?

- | | |
|------------------------|-----------------------------------|
| A gravitational force | B electrostatic force |
| C elastic spring force | D kinetic frictional force |

Solution: The work done by kinetic friction depends on the length of the path, so it is non-conservative.

MCQ 4.3.4 The work done by gravity along a horizontal step of a curved path is:

- | | |
|---------------|----------------|
| A zero | B mgh |
| C $-mgh$ | D $mg\Delta y$ |

Solution: The weight is perpendicular to a horizontal displacement, and $\cos 90^\circ = 0$

MCQ 4.3.5 The work done by gravity in raising a body of mass m through a height h is:

- | | |
|---------|--------------------|
| A mgh | B $-mgh$ |
| C zero | D $\frac{1}{2}mgh$ |

Solution: The weight acts downward while the displacement is upward, so the angle is 180° and the work is negative.

MCQ 4.3.6 Which one of the following is a non-conservative force?

- | | |
|---------------------------------------|-----------------------|
| A elastic spring force | B electrostatic force |
| C propulsion force of a rocket | D gravitational force |

Solution: The work done by the propulsion force depends on the path of motion, so it is non-conservative.

MCQ 4.3.7 The total work done by a non-conservative force in a closed path is:

- | | |
|-------------------|-------------------|
| A always zero | B not zero |
| C always positive | D equal to mgh |

Solution: Friction and the other non-conservative forces keep doing negative work all round the loop, so the total does not cancel.

Topic 4.4 Power

Need for the Idea

The definition of work says nothing about the time taken, so it does not tell whether the same amount of work is done in one second or in one hour. The rate at which work is done is often of more practical interest.

Power

Power is the measure of the rate at which work is being done.

Average Power

If work ΔW is done in a time interval Δt , the average power during that interval is defined as the work divided by the time.

$$P_{av} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power

If the work is expressed as a function of time, the instantaneous power at any instant is the limit of the same ratio as the interval becomes vanishingly small.

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$$

Power in Terms of Force and Velocity

The work done in a short interval is the scalar product of the force and the displacement.

$$\Delta W = \vec{F} \cdot \Delta \vec{d}$$

Putting it in the definition of the power

$$P = \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$P = \vec{F} \cdot \frac{\Delta \vec{d}}{\Delta t}$$

The ratio of the displacement to the time is the velocity of the body.

$$\frac{\Delta \vec{d}}{\Delta t} = \vec{v}$$

Result

$$P = \vec{F} \cdot \vec{v}$$

Meaning

Power may therefore also be defined as the scalar product of the force and the velocity.

SI Unit

The SI unit of power is the watt, defined as one joule of work done in one second.

$$1 \text{ W} = 1 \text{ J s}^{-1}$$

Watt-Second

In electrical measurement the unit of work is sometimes expressed as the watt-second, which is the same as the joule.

Kilowatt-Hour

The commercial unit of electrical energy is the kilowatt-hour. One kilowatt-hour is the work done in one hour by an agency whose power is one kilowatt.

Derivation

$$1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ s}$$

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J} = 3.6 \text{ MJ}$$

Approximate Powers

- A jumbo jet aircraft delivers about $1.3 \times 10^8 \text{ W}$.
- A car at 90 km h^{-1} delivers about $1.1 \times 10^5 \text{ W}$.
- An electric heater takes about $2 \times 10^3 \text{ W}$.
- A coloured television takes about 120 W .
- A flash light of two cells takes about 1.5 W .
- A pocket calculator takes about $7.5 \times 10^{-4} \text{ W}$.

Book Examples

Example 4.4.1 A 70 kg man runs up a flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Given Data

$$m = 70 \text{ kg}$$

$$h = 4.5 \text{ m}$$

$$t = 4.0 \text{ s}$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find Power output $P = ?$

Formula

The work done is done against the weight of the man through the vertical height.

$$W = mgh$$

$$P = \frac{W}{t}$$

$$P = \frac{mgh}{t}$$

Solution

Putting the values in the formula

$$P = \frac{70 \times 9.8 \times 4.5}{4.0} = 771.75 \text{ W}$$

Result $P = 7.7 \times 10^2 \text{ W}$

Short Questions**SQ 4.4.1 Define power and give its SI unit.****Power**

Power is the measure of the rate at which work is being done.

Mathematically $P_{av} = \frac{\Delta W}{\Delta t}$

SI Unit $\text{watt} = \text{J s}^{-1}$

Definition of a Watt

One watt is the power of an agency which does one joule of work in one second.

SQ 4.4.2 Show that power is the scalar product of force and velocity.

Work in a Short Interval $\Delta W = \vec{F} \cdot \Delta \vec{d}$

Putting it in the Definition of Power

$$P = \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t} = \vec{F} \cdot \frac{\Delta \vec{d}}{\Delta t}$$

The ratio of the displacement to the time is the velocity.

$$\frac{\Delta \vec{d}}{\Delta t} = \vec{v}$$

Result

$$P = \vec{F} \cdot \vec{v}$$

So power is the scalar product of the force and the velocity.

SQ 4.4.3 Define the kilowatt-hour and express it in joules.**Kilowatt-Hour**

One kilowatt-hour is the work done in one hour by an agency whose power is one kilowatt. It is the commercial unit of electrical energy.

Derivation $1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ s}$

Result

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J} = 3.6 \text{ MJ}$$

SQ 4.4.4 Distinguish between average power and instantaneous power.

Average Power	Instantaneous Power
It is the total work divided by the <u>whole time</u> taken.	It is the power at one particular <u>instant</u> .
$P_{av} = \frac{\Delta W}{\Delta t}$	$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$
It tells nothing about how the rate changed during the interval.	It tells how fast work is being done at that moment.

SQ 4.4.5 A pump lifts 200 kg of water to a height of 10 m in 20 s. Find its power.**Given Data**

$$m = 200 \text{ kg}$$

$$h = 10 \text{ m}$$

$$t = 20 \text{ s}$$

Formula

$$P = \frac{mgh}{t}$$

Solution

$$P = \frac{200 \times 9.8 \times 10}{20} = \frac{19600}{20} = 980 \text{ W}$$

Result $P = 980 \text{ W}$

MCQ 4.4.1 The SI unit of power is the:

- | | |
|---------------|-----------------|
| A joule | B newton |
| C watt | D kilowatt-hour |

Solution: One watt is one joule of work done in one second.

MCQ 4.4.2 One kilowatt-hour is equal to:

- | | |
|---|-------------------------------|
| A $3.6 \times 10^3 \text{ J}$ | B $3.6 \times 10^5 \text{ J}$ |
| C $3.6 \times 10^6 \text{ J}$ | D $3.6 \times 10^9 \text{ J}$ |

Solution: $1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \text{ MJ}$

MCQ 4.4.3 Power may also be expressed as the:

- | | |
|---|--|
| A scalar product of force and velocity | B vector product of force and velocity |
| C product of force and displacement | D ratio of force to velocity |

Solution: $P = \Delta W / \Delta t = \vec{F} \cdot \Delta \vec{d} / \Delta t = \vec{F} \cdot \vec{v}$

MCQ 4.4.4 A 100 W bulb consumes energy in one minute equal to:

- | | |
|---------|-----------------|
| A 100 J | B 6000 J |
| C 600 J | D 60 J |

Solution: $W = Pt = 100 \times 60 = 6000 \text{ J}$

MCQ 4.4.5 If the same amount of work is done in half the time, the power becomes:

- | | |
|-----------------|--------------|
| A half | B one fourth |
| C double | D unchanged |

Solution: Power is inversely proportional to the time for a fixed amount of work.

MCQ 4.4.6 A body moves with a constant speed of 5 m s^{-1} against a resistive force of 20 N. The power expended is:

- | | |
|----------------|---------|
| A 4 W | B 25 W |
| C 100 W | D 400 W |

Solution: $P = Fv = 20 \times 5 = 100 \text{ W}$

MCQ 4.4.7 The watt-second is another name for the:

- | | |
|-----------------|----------------|
| A watt | B joule |
| C kilowatt-hour | D newton |

Solution: A watt multiplied by a second is one joule of work.

Topic 4.5 Energy, Kinetic Energy and Absolute Potential Energy**Energy**

The energy of a body is its capacity to do work. There are two basic forms of mechanical energy.

Two Forms

- Kinetic energy, the energy possessed by a body due to its motion.
- Potential energy, the energy possessed by a body due to its changed position.

LQ 1 starts here — Define K.E. and derive an expression for the same.

Kinetic Energy

Consider a car running with a constant speed on a road. If its engine is switched off it still covers some distance before stopping, and as long as it moves it does work against the force of friction f of the road. The work done by the car is therefore fd , and this work is done because of its motion.

Acceleration of the Car

As the car slows down and stops, its acceleration is negative because the friction acts opposite to the motion.

$$f = -ma$$

$$a = -\frac{f}{m}$$

Using the Third Equation of Motion

The initial velocity is v , the final velocity is zero and the distance covered is d

$$2ad = v_f^2 - v_i^2$$

Putting the value of the acceleration in it

$$2\left(-\frac{f}{m}\right)d = 0 - v^2$$

$$-\frac{2fd}{m} = -v^2$$

$$fd = \frac{1}{2}mv^2$$

Kinetic Energy

The ability of a body to do work because of its motion is its kinetic energy, and it is equal to the work fd the body can do.

$$K.E. = \frac{1}{2}mv^2$$

Unit

Kinetic energy is equal to a quantity of work, so its unit is the unit of work, the joule.

LQ 1 ends here

Potential Energy

The potential energy is possessed by a body because of its position in a force field, such as the gravitational field, or because of its constrained state.

Elastic Potential Energy

The energy stored in a compressed or stretched spring is possessed by the spring because of its state, and this form is called elastic potential energy.

LQ 4 starts here — What is meant by absolute P.E.? Derive an expression for absolute P.E.

Absolute PE

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing it from that position to infinity, where the force of gravity becomes zero.

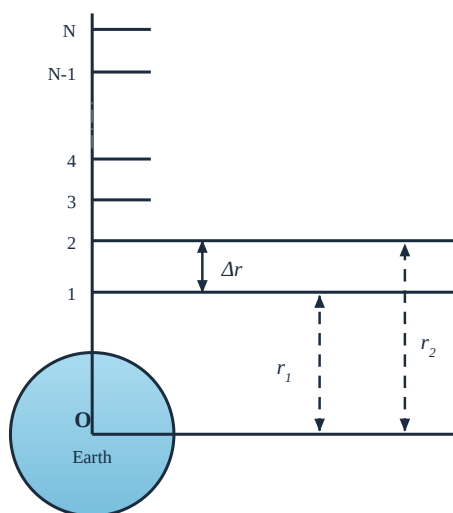
Explanation

The relation mgh holds only near the surface of the Earth, where the gravitational force is nearly constant. Over a large distance the force varies inversely as the square of the distance from the centre of the Earth, so it cannot be treated as constant.

Divide the Distance

The distance is divided into small steps of length Δr , each so short that the force may be taken as constant over it, and the work done in all the steps is added.

Figure



Derivation

The force is evaluated at the middle of the first step, whose ends are at the distances r_1 and r_2 from the centre of the Earth.

$$r = \frac{r_1 + r_2}{2}$$

$$r_2 - r_1 = \Delta r$$

$$r_2 = r_1 + \Delta r$$

$$r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = r_1 + \frac{\Delta r}{2}$$

Squaring both sides of it

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + 2 \times r_1 \times \frac{\Delta r}{2} + \frac{(\Delta r)^2}{4}$$

$$r^2 = r_1^2 + r_1 \Delta r + \frac{(\Delta r)^2}{4}$$

The step is so small that the square of it is negligible against the rest.

$$(\Delta r)^2 \approx 0$$

$$r^2 = r_1^2 + r_1 \Delta r$$

$$r^2 = r_1(r_1 + \Delta r)$$

$$r^2 = r_1 r_2$$

$$F = -\frac{GMm}{r^2}$$

$$F = -\frac{GMm}{r_1 r_2}$$

F on 1st Step

W on 1st Step

The force is constant over the step and opposite to the displacement.

$$W_{1 \rightarrow 2} = F \Delta r$$

$$W_{1 \rightarrow 2} = -\frac{GMm}{r_1 r_2} (r_2 - r_1)$$

$$W_{1 \rightarrow 2} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

W in next steps

Every step gives a difference of the same kind, so adding them all leaves only the two end terms.

$$W_{2 \rightarrow 3} = -GMm \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$$

$$W_{(N-1) \rightarrow N} = -GMm \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$$

$$W_{1 \rightarrow N} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_N} \right)$$

To infinity

The point N is taken at an infinite distance from the Earth, where the second term vanishes.

$$r_N = \infty$$

$$\frac{1}{r_N} = 0$$

$$W_{1 \rightarrow \infty} = -\frac{GMm}{r_1}$$

General Form

This total work is by definition the absolute potential energy of the body at a distance r from the centre of the Earth.

$$U = -\frac{GMm}{r}$$

On Earth's Surface

Putting the radius of the Earth for the distance gives the value at the surface.

$$r = R$$

$$U_g = -\frac{GMm}{R}$$

Negative Sign

The negative sign shows that the gravitational field of the Earth is attractive for the mass m . The expression gives the energy required to take the body out of the gravitational field of the Earth, where its potential energy is zero.

Relation with height

As r increases, U becomes less negative, that is the potential energy increases. So raising a body above the surface of the Earth increases its potential energy, and at an infinite distance it becomes zero.

LQ 4 ends here

Book Examples

Example 4.5.1 A car weighing 18620 N is running with a speed of 16 m s^{-1} . The brakes are applied and it is brought to rest in a distance of 80 m. Determine the average force of friction acting on it.

Given Data

$$w = 18620 \text{ N}$$

$$v = 16 \text{ m s}^{-1}$$

$$d = 80 \text{ m}$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find

Average force of friction $f = ?$

Formula

The kinetic energy of the car is equal to the work done by it against friction before stopping.

$$\frac{1}{2}mv^2 = fd$$

$$f = \frac{mv^2}{2d}$$

Solution

The mass of the car is its weight divided by the acceleration due to gravity.

$$m = \frac{w}{g} = \frac{18620}{9.8} = 1900 \text{ kg}$$

Putting the values in the formula

$$f = \frac{1900 \times (16)^2}{2 \times 80} = \frac{486400}{160} = 3040 \text{ N}$$

Result $f = 3040 \text{ N}$

Short Questions

SQ 4.5.1 Define energy and name its two basic forms.

Energy

The energy of a body is its capacity to do work.

Two Basic Forms

- Kinetic energy, possessed by a body because of its motion.
- Potential energy, possessed by a body because of its changed position.

Mechanical Energy

Both of these are kinds of mechanical energy, and both are measured in joules.

SQ 4.5.2 What is elastic potential energy? Give an example.

Elastic Potential Energy

It is the energy possessed by a body because of its constrained state rather than its position in a field.

Example

The energy stored in a compressed or a stretched spring. The spring can do work as it returns to its natural length, so it holds energy while it is deformed.

SQ 4.5.3 Why is the absolute gravitational potential energy of a body negative?

Zero Taken at Infinity

The zero of absolute potential energy is taken at infinity, where the gravitational force vanishes. Every point nearer the Earth has a lower energy than that.

$$U = -\frac{GMm}{r}$$

Meaning of the Sign

The negative sign shows that the gravitational field of the Earth is attractive for the mass m , so work has to be done to take the body out of the field.

SQ 4.5.4 Why can the relation mgh not be used for large distances from the Earth?

Gravity Not Constant Far from the Earth

The relation mgh assumes that the gravitational force is constant, which is true only near the surface of the Earth.

Over a Large Distance

The force varies inversely as the square of the distance from the centre of the Earth, so it cannot be taken outside the calculation as a constant.

Way Out

The distance is divided into small steps over each of which the force may be treated as constant.

SQ 4.5.5 What happens to the potential energy of a body as it is raised higher above the Earth?

Absolute Potential Energy $U = -\frac{GMm}{r}$

Effect of Raising the Body

As r increases, U becomes less negative, that is, the potential energy increases.

At Infinity

If the body is taken to an infinite distance, its potential energy becomes zero, which is the largest value it can have.

SQ 4.5.6 Find the kinetic energy of a body of mass 2 kg moving with a speed of 10 m s^{-1} .

Given Data $m = 2 \text{ kg}$

$$v = 10 \text{ m s}^{-1}$$

Formula $K.E. = \frac{1}{2}mv^2$

Solution

$$K.E. = \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ J}$$

Result $K.E. = 100 \text{ J}$

SQ 4.5.7 On what factors does the kinetic energy of a body depend, and how?

Factors Kinetic Energy Depends On $K.E. = \frac{1}{2}mv^2$

Mass

The kinetic energy is directly proportional to the mass, so doubling the mass doubles it.

Speed

It is directly proportional to the square of the speed, so doubling the speed makes the kinetic energy four times as great.

Direction

It does not depend on the direction of motion, because kinetic energy is a scalar.

MCQ 4.5.1 The kinetic energy of a body of mass m moving with a speed v is:

A mv

B mv^2

C $\frac{1}{2}mv^2$

D $\frac{1}{2}mv$

Solution: The work a moving body can do before stopping comes to $\frac{1}{2}mv^2$

MCQ 4.5.2 If the speed of a body is doubled, its kinetic energy becomes:

- A double B half
C **four times** D unchanged

Solution: The kinetic energy is proportional to the square of the speed.

MCQ 4.5.3 The absolute gravitational potential energy of a body at a distance r from the centre of the Earth is:

- A $\frac{GMm}{r}$ B $-\frac{GMm}{r}$
C $-\frac{GMm}{r^2}$ D mgr

Solution: The zero of potential energy is taken at infinity, so every finite distance gives a negative value.

MCQ 4.5.4 The absolute potential energy of a body on the surface of the Earth is:

- A zero B mgh
C $-\frac{GMm}{R^2}$ D $-\frac{GMm}{R}$

Solution: Putting the radius of the Earth for the distance in $U = -GMm/r$ gives the value at the surface.

MCQ 4.5.5 The negative sign in the expression for absolute potential energy shows that the gravitational field is:

- A repulsive B **attractive**
C uniform D zero

Solution: Work has to be done on the body to take it out of the field, so its energy inside the field is below the zero at infinity.

MCQ 4.5.6 The energy stored in a compressed spring is called:

- A kinetic energy B gravitational potential energy
C **elastic potential energy** D thermal energy

Solution: The spring holds the energy because of its constrained state, not because of its position in a field.

MCQ 4.5.7 If the momentum of a body is doubled, its kinetic energy becomes:

- A double B half
C **four times** D unchanged

Solution: $K.E. = p^2/2m$, so the kinetic energy goes as the square of the momentum at a fixed mass.

MCQ 4.5.8 As a body is taken farther from the Earth, its absolute potential energy:

- A **increases** B decreases
C stays the same D becomes more negative

Solution: $U = -GMm/r$ becomes less negative as r grows, so the potential energy increases towards zero.

Topic 4.6 Escape Velocity

LQ 6 starts here — Define escape velocity. Show that an expression for escape velocity can be expressed as $v = \sqrt{2Rg}$, where R and g denote the radius of the Earth and the acceleration due to gravity respectively. Also find its numerical value near the surface of the Earth.

Everyday Observation

An object projected upward comes back to the ground after rising to a certain height, because the force of gravity acts downward. With a greater initial velocity it rises higher before coming back.

The Limiting Case

If the initial velocity is increased further, a stage comes when the object will not return to the ground at all. It escapes from the influence of gravity.

Escape Velocity

The initial velocity of an object with which it goes out of the gravitational field of the Earth is known as its escape velocity.

Initial Kinetic Energy

The escape velocity corresponds to the initial kinetic energy given to the body, which carries it to an infinite distance from the surface of the Earth.

$$K.E._i = \frac{1}{2}mv_{esc}^2$$

Increase in Potential Energy

The work done in lifting a body from the surface of the Earth to an infinite distance is equal to the increase in its potential energy.

$$\Delta U = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Condition for Escape

The body escapes out of the gravitational field if its initial kinetic energy is equal to this increase in potential energy.

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$$

$$v_{esc}^2 = \frac{2GM}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

In Terms of the Acceleration Due to Gravity

The acceleration due to gravity at the surface of the Earth is itself fixed by the mass and the radius of the Earth.

$$g = \frac{GM}{R^2}$$

$$gR = \frac{GM}{R}$$

Result

$$v_{esc} = \sqrt{2gR}$$

Numerical Value

Putting the acceleration due to gravity and the radius of the Earth in the result gives the escape velocity at the surface of the Earth.

$$v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} \approx 11 \text{ km s}^{-1}$$

Independent of Mass

The mass m of the body cancels out of the derivation, so the escape velocity is the same for every body and depends only on the mass and the radius of the planet.

Some Escape Speeds

The escape speed of a heavenly body depends only on its own mass and radius, so every planet has its own.

Some Escape speeds (km s^{-1})	
Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

LQ 6 ends here

Short Questions

SQ 4.6.1 Why does the escape velocity not depend on the mass of the body projected?

Mass of the Body Cancels Out

The mass of the body appears on both sides of the condition for escape and cancels out.

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Result

The escape velocity depends only on the mass and the radius of the planet, so a pebble and a spacecraft need the same escape velocity.

SQ 4.6.2 What would happen to the escape velocity of the Earth if its radius were doubled, its mass staying the same?

Escape Velocity and the Radius

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Effect

The escape velocity is inversely proportional to the square root of the radius.

$$v_{esc} \propto \frac{1}{\sqrt{R}}$$

Result

Doubling the radius would make the escape velocity $1/\sqrt{2}$ times its present value, that is about **7.9 km s⁻¹**.

SQ 4.6.3 Why is the escape velocity of the Moon much less than that of the Earth?

Smaller Mass of the Moon

The escape velocity depends on the mass and the radius of the body.

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

The Moon

The mass of the Moon is far smaller than that of the Earth, so the gravitational pull at its surface is much weaker.

Result

Its escape velocity is only about 2.4 km s⁻¹ against 11.2 km s⁻¹ for the Earth, which is why the Moon has no atmosphere.

SQ 4.6.4 What is the value of the escape velocity at the surface of the Earth?

Escape Velocity at the Surface

$$v_{esc} = \sqrt{2gR}$$

Substitution

$$v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

Result

$$v_{esc} \approx 11 \text{ km s}^{-1}$$

Meaning

A body thrown from the surface of the Earth with this speed will never return to the ground.

SQ 4.6.5 Why does an object thrown upward normally come back to the ground?

Pull of Gravity on the Way Up

The force of gravity acts downward all the way, so the object is decelerated, stops and falls back.

Energy View

The kinetic energy given to it is less than the increase in potential energy needed to reach infinity.

$$\frac{1}{2}mv^2 < \frac{GMm}{R}$$

Result

Only if the initial speed reaches the escape velocity does the object leave the gravitational field of the Earth.

MCQ 4.6.1 The escape velocity at the surface of the Earth is about:

- | | |
|--------------------------|-------------------------------|
| A 8 km s ⁻¹ | B 11 km s⁻¹ |
| C 2.4 km s ⁻¹ | D 25 km s ⁻¹ |

Solution: $v_{esc} = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} \approx 11 \text{ km s}^{-1}$

MCQ 4.6.2 The escape velocity of a planet is given by:

- | | |
|---------------|----------------------------------|
| A $\sqrt{gR}$ | B $\sqrt{2gR}$ |
| C $2gR$ | D $\sqrt{\frac{gR}{2}}$ |

Solution: Equating the initial kinetic energy to the increase in potential energy gives

$$v_{esc} = \sqrt{2GM/R} = \sqrt{2gR}$$

MCQ 4.6.3 The escape velocity of a body does NOT depend on:

- | | |
|-------------------------------|--|
| A the mass of the planet | B the radius of the planet |
| C the mass of the body | D the acceleration due to gravity at the surface |

Solution: The mass of the body cancels from both sides of the energy condition.

MCQ 4.6.4 If the mass of a planet is doubled, its radius staying the same, the escape velocity becomes:

- | | |
|--------------------------------------|--------------|
| A double | B half |
| C $\sqrt{2}$ times | D four times |

Solution: $v_{esc} \propto \sqrt{M}$, so doubling the mass multiplies the escape velocity by $\sqrt{2}$

MCQ 4.6.5 The escape velocity of the Moon is about:

- | | |
|--------------------------------|--------------------------|
| A 2.4 km s⁻¹ | B 4.3 km s ⁻¹ |
| C 11.2 km s ⁻¹ | D 37 km s ⁻¹ |

Solution: The Moon has a much smaller mass than the Earth, so its escape velocity is far lower, which is why it holds no atmosphere.

MCQ 4.6.6 A body just escapes from the gravitational field of the Earth when its initial kinetic energy is:

- | | |
|---|--|
| A greater than the increase in its potential energy | B equal to the increase in its potential energy |
| C less than the increase in its potential energy | D zero |

Solution: At exactly the escape velocity the kinetic energy supplied is exactly what is needed to reach infinity, where the body arrives with zero speed.

Topic 4.7 Work-Energy Theorem

LQ 5 starts here — State and explain work-energy theorem in a resistive medium.

Idea

Whenever work is done on a body it increases the energy of the body. The theorem states exactly how much.

Setting Up the Derivation

A force F acts on a body of mass m which is initially moving with a velocity v_i . The force acts through a distance d and increases the velocity to v_f , so the acceleration produced follows from the third equation of motion.

$$2ad = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2d}$$

From the Second Law of Motion $F = ma$

$$F = m \left(\frac{v_f^2 - v_i^2}{2d} \right)$$

Work Done by the Force

The work done is the product of this force and the distance through which it acts.

$$W = Fd$$

$$W = m \left(\frac{v_f^2 - v_i^2}{2d} \right) d$$

$$W = \frac{m(v_f^2 - v_i^2)}{2}$$

Result

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Statement

The change in the kinetic energy of an object is equal to the work done on it by a net force. This is also known as the work-energy principle.

In Words $W = (K.E.)_f - (K.E.)_i$

Scope of the Theorem

The theorem is applicable for any direction of the force relative to the displacement, and it remains valid even if the force varies from point to point.

Negative Work

An object with kinetic energy can do work by pushing or pulling on another object. In that case the work is taken as negative and the kinetic energy of the object decreases.

Work-Energy Theorem in a Resistive Medium

A body falling through a height h in a medium which offers a resistive force f does work against that force as well, so not all of the potential energy appears as kinetic energy.

Work Done Against the Resistance

$$W_f = fh$$

Energy Available

The potential energy left over after the work against the resistance has been paid for is what turns into kinetic energy.

$$mgh - fh = \frac{1}{2}mv^2$$

Statement in a Resistive Medium

The loss in potential energy is equal to the gain in kinetic energy plus the work done against friction.

Use

The relation is what lets the average resistive force of the air be calculated from the height of a fall and the speed with which the body strikes the ground.

IQ 5 ends here

Book Examples

Example 4.7.1 A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of the motorcycle is 30 kg. At the top of the slope the speed of the motorcycle is 3.2 m s^{-1} . If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?

Given Data

$$m = 60 + 30 = 90 \text{ kg}$$

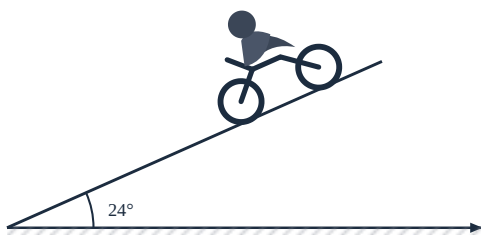
$$\theta = 24^\circ$$

$$v_i = 3.2 \text{ m s}^{-1}$$

$$f = 100 \text{ N}$$

$$d = 72 \text{ m}$$

Diagram



To Find

Speed $v_f = ?$ after a distance of 72 m

Formula

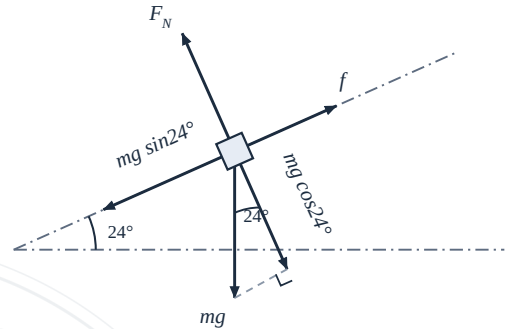
The normal force is balanced by the component of the weight perpendicular to the slope, so the net force along the slope is the component of the weight along it, less the friction.

$$F = mg \sin 24^\circ - f$$

$$W = Fd$$

$$W = (K.E.)_f - (K.E.)_i$$

Forces on the Slope



Solution

Net Force

$$F = (90 \times 9.8 \times 0.4) - 100 = 352.8 - 100 = 252.8 \text{ N}$$

Work Done

$$W = 252.8 \text{ N} \times 72 \text{ m} = 18201.6 \text{ J}$$

Applying the Work-Energy Theorem

The work is positive, so the final kinetic energy is the work plus the initial kinetic energy.

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$45 v_f^2 = 18201.6 + \frac{1}{2} \times 90 \times (3.2)^2 = 18201.6 + 460.8 = 18662.4$$

$$v_f^2 = \frac{18662.4}{45} = 414.7 \text{ m}^2 \text{ s}^{-2}$$

Result

$$v_f = 20.4 \text{ m s}^{-1}$$

Short Questions

SQ 4.7.1 State the work-energy theorem.

Statement

The change in the kinetic energy of an object is equal to the work done on it by a net force.

Mathematically

$$W = (K.E.)_f - (K.E.)_i$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Other Name

It is also known as the work-energy principle.

SQ 4.7.2 What does it mean if the work done on a body by the net force comes out negative?

Negative Work Means a Loss of K.E.

A negative work means the final kinetic energy is less than the initial kinetic energy.

$$W = (K.E.)_f - (K.E.)_i < 0$$

Physical Picture

The force opposes the motion, so the body slows down. Braking a car and a body rising against gravity are both examples.

SQ 4.7.3 Is the work-energy theorem valid when the force varies from point to point? Justify.

Yes, it is valid

The theorem holds for any direction of the force relative to the displacement and even when the force varies from point to point.

Reason

The work of a variable force is found as the area under its force-displacement curve, and whatever that work comes to, it still shows up as the change in kinetic energy.

SQ 4.7.4 A body of mass 2 kg is speeded up from 3 m s^{-1} to 5 m s^{-1} . Find the work done on it.

Given Data

$$m = 2 \text{ kg}$$

$$v_i = 3 \text{ m s}^{-1}$$

$$v_f = 5 \text{ m s}^{-1}$$

Formula $W = \frac{1}{2}m(v_f^2 - v_i^2)$

Solution $W = \frac{1}{2} \times 2 \times (25 - 9) = 16 \text{ J}$

Result $W = 16 \text{ J}$

SQ 4.7.5 Why does not all of the potential energy appear as kinetic energy when a body falls through a resistive medium?

Work Against the Resistance

A part of the potential energy is spent in doing work against the resistive force of the medium.

$$W_f = fh$$

Energy Left Over

Only the remainder is converted into kinetic energy.

$$mgh - fh = \frac{1}{2}mv^2$$

No Loss of Energy

The energy taken by the resistance is not destroyed but appears as heat and sound.

MCQ 4.7.1 According to the work-energy theorem, the work done by the net force on a body equals the change in its:

A momentum B potential energy

C **kinetic energy** D power

Solution: $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, which is the change in kinetic energy.

MCQ 4.7.2 If the work done on a moving body by the net force is negative, its speed:

A increases B **decreases**

C stays the same D becomes zero at once

Solution: A negative work means the final kinetic energy is smaller than the initial one.

MCQ 4.7.3 The work-energy theorem holds:

A only for a constant force B only when the force is along the displacement

C only for a body at rest D **for a variable force and any direction of the force**

Solution: The theorem places no restriction on how the force varies or on its direction relative to the displacement.

MCQ 4.7.4 A body of mass 1 kg is accelerated from rest to 4 m s^{-1} . The work done on it is:

A 4 J B **8 J**

C 16 J D 2 J

Solution: $W = \frac{1}{2} \times 1 \times (16 - 0) = 8 \text{ J}$

MCQ 4.7.5 For a body falling through a resistive medium, $mgh - fh$ is equal to:

A mgh B fh

C $\frac{1}{2}mv^2$ D zero

Solution: What is left of the potential energy after the work against the resistance has been paid for appears as kinetic energy.

MCQ 4.7.6 When a body moves with a constant speed, the net work done on it is:

A **zero** B $\frac{1}{2}mv^2$

C mgh D negative

Solution: A constant speed means no change in kinetic energy, so the net work is zero.

MCQ 4.7.7 The work-energy theorem is also known as the:

A law of conservation of momentum B **work-energy principle**

C law of inertia D principle of moments

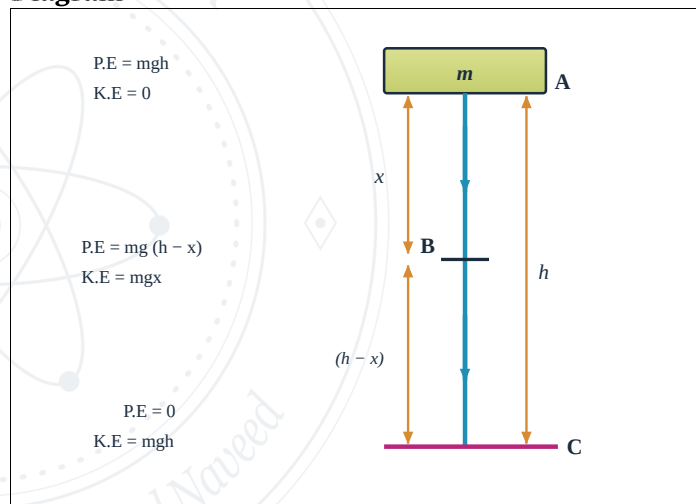
Solution: The two names refer to the same statement relating work to the change in kinetic energy.

Topic 4.8 Interconversion of Potential Energy and Kinetic Energy

Setting

Consider a body of mass m at rest at a height h above the surface of the Earth. It is released and falls freely, and air friction is ignored to begin with.

Diagram



At the Top, Position A

The body is at rest at the full height, so it has potential energy only.

$$P.E. = mgh$$

$$K.E. = 0$$

At an Intermediate Point B

The body has fallen through a distance x , so its height above the ground is $h - x$

$$P.E. = mg(h - x)$$

Velocity at B

The velocity at B follows from the third equation of motion with an initial velocity of zero.

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

Kinetic Energy at B

$$K.E. = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2gx) = mgx$$

Total Energy at B

Adding the two energies at the intermediate point gives back the energy the body started with.

$$E = mg(h - x) + mgx$$

$$E = mgh$$

At the Ground, Position C

Just before the body strikes the Earth its height is zero, so it has kinetic energy only.

$$v_C^2 = v_i^2 + 2gh$$

$$v_C^2 = 0 + 2gh = 2gh$$

$$K.E. = \frac{1}{2}m(2gh) = mgh$$

$$P.E. = 0$$

Conclusion

At the ground the kinetic energy is equal to the original potential energy of the body. As the body falls its velocity and hence its kinetic energy increase, while its height and hence its potential energy decrease by the same amount.

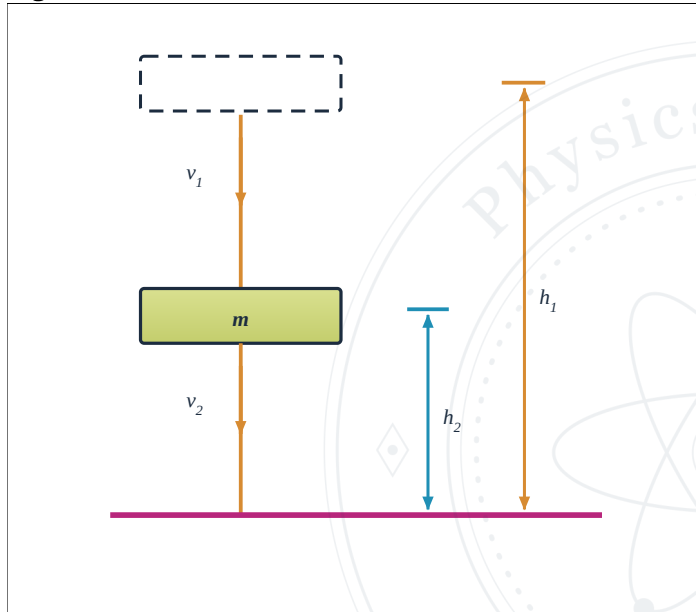
Law of Conservation of Energy

$$\text{Loss in } P.E. = \text{Gain in } K.E.$$

Between Any Two Heights

If v_1 and v_2 are the velocities of the body at the heights h_1 and h_2 , the same statement takes a general form.

Figure



$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

Condition

This result is true only when the frictional force is not considered.

With Friction Present

If a frictional force f acts during the downward motion, a part of the potential energy is used in doing work against friction, equal to fh . Only the remainder is converted into kinetic energy.

$$mgh - fh = \frac{1}{2}mv^2$$

General Statement

Loss in potential energy is equal to the gain in kinetic energy plus the work done against friction.

$$mgh = \frac{1}{2}mv^2 + fh$$

Conversely

For a body moving upward, the loss of kinetic energy is equal to the gain in potential energy plus the work done against friction.

Significance

Energy is never lost in either case. What friction takes is not destroyed but changed into heat and sound, so the total energy of the system stays the same.

Book Examples

Example 4.8.1 A car weighing 1100 kg is moving with a velocity of 12 m s^{-1} . When it is at the point P its engine stops. If the frictional force is 120 N, what will be its velocity at the point Q, which is 1.5 m higher and 24 m along the road? How far beyond Q will it go before coming to rest?

Given Data

$$m = 1100 \text{ kg}$$

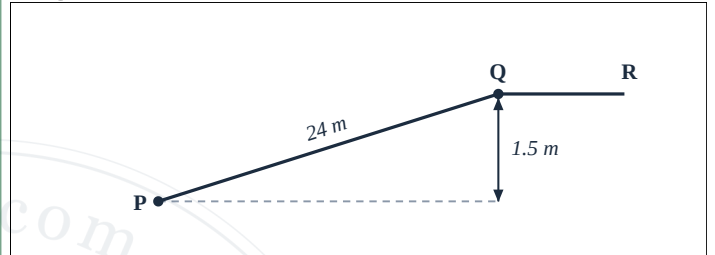
$$v_P = 12 \text{ m s}^{-1}$$

$$h = 1.5 \text{ m}$$

$$d = 24 \text{ m}$$

$$f = 120 \text{ N}$$

Diagram



To Find

Velocity at the point Q is $v_Q = ?$ and the further distance is $s = ?$

Formula

The kinetic energy at P is partly converted into potential energy and partly used in doing work against friction.

$$\frac{1}{2}m(v_P^2 - v_Q^2) = mgh + fd$$

Solution

Putting the values in the formula

$$\frac{1}{2} \times 1100 (144 - v_Q^2) = (1100 \times 9.8 \times 1.5) + (120 \times 24)$$

$$550 (144 - v_Q^2) = 16170 + 2880 = 19050$$

$$144 - v_Q^2 = \frac{19050}{550} = 34.6$$

$$v_Q^2 = 144 - 34.6 = 109.4 \text{ m}^2 \text{ s}^{-2}$$

$$v_Q = \sqrt{109.4} = 10.5 \text{ m s}^{-1}$$

Distance Beyond Q

Beyond Q the whole of the remaining kinetic energy is spent against friction.

$$\frac{1}{2}mv_Q^2 = fs$$

$$s = \frac{\frac{1}{2} \times 1100 \times 109.4}{120} = \frac{60170}{120}$$

$$= 501 \text{ m}$$

Result

$$v_Q = 10.5 \text{ m s}^{-1}$$

$$s \approx 501 \text{ m}$$

Example 4.8.2 An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s^{-1} , calculate the average frictional force of the air.

Given Data

$$m = 3 \text{ kg}$$

$$h = 15 \text{ m}$$

$$v = 16 \text{ m s}^{-1}$$

$$g = 9.8 \text{ m s}^{-2}$$

To FindAverage frictional force of the air $f = ?$ **Formula**

The loss in potential energy is the gain in kinetic energy plus the work done against friction.

$$mgh = \frac{1}{2}mv^2 + fh$$

$$f = \frac{mgh - \frac{1}{2}mv^2}{h}$$

Solution

Putting the values in the formula

$$3 \times 9.8 \times 15 = \frac{1}{2} \times 3 \times (16)^2 + f \times 15$$

$$441 = 384 + 15f$$

$$f = \frac{441 - 384}{15} = \frac{57}{15} = 3.8 \text{ N}$$

Result $f = 3.8 \text{ N}$ **Short Questions****SQ 4.8.1** A body falls freely from a height h . Show that at the ground its kinetic energy is equal to its initial potential energy.**At the Top** $P.E. = mgh$

$$K.E. = 0$$

Velocity at the Ground $v^2 = 0 + 2gh$ **Kinetic Energy at the Ground**

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh)$$

Result

$$K.E. = mgh$$

So at the ground the kinetic energy is exactly the original potential energy of the body.**SQ 4.8.2** What is the total energy of a freely falling body at any point of its path?**Total Energy**

At every point the sum of the potential and the kinetic energy is the same and equal to the energy the body started with.

At a Distance x Below the Start

$$P.E. = mg(h - x)$$

$$K.E. = mgx$$

$$E = mg(h - x) + mgx$$

Result

$$E = mgh$$

So the total energy stays constant throughout the fall.**SQ 4.8.3** State the law of conservation of energy for a freely falling body.**Statement**For a body falling freely, the loss in potential energy is equal to the gain in kinetic energy, the total remaining unchanged.**Mathematically**

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

Condition

The result holds only when the frictional force of the air is not considered.

SQ 4.8.4 How is the relation "loss in P.E. = gain in K.E." modified when friction is present?**With Friction**A part of the potential energy is used in doing work against friction, equal to fh , so only the rest appears as kinetic energy.

$$mgh - fh = \frac{1}{2}mv^2$$

General StatementLoss in potential energy is equal to the gain in kinetic energy plus the work done against friction.

$$mgh = \frac{1}{2}mv^2 + fh$$

SQ 4.8.5 A ball dropped on the floor does not rebound to the same height. Why?**Energy Spent in the Impact**A part of the mechanical energy of the ball is spent against air resistance and is used up in deforming the ball and the floor during the impact.**What Becomes of It**

That energy is changed into heat and sound, so it is no longer available as kinetic energy.

Result

The ball rises to a smaller height on each bounce, though the total energy of the ball and its surroundings is unchanged.

MCQ 4.8.1 At the highest point of the path of a freely falling body released from rest:

A the kinetic energy is maximum	B the potential energy is maximum and the kinetic energy is zero
C both energies are equal	D both energies are zero

Solution: The body is at rest at the full height, so all of its energy is potential.**MCQ 4.8.2** A body falls freely through a distance x from rest. Its kinetic energy at that point is:

A mgh	B mgx
C $mg(h - x)$	D $\frac{1}{2}mgx$

Solution: $v^2 = 2gx$, so $K.E. = \frac{1}{2}m(2gx) = mgx$ **MCQ 4.8.3** For a freely falling body the sum of the potential and the kinetic energy at every point is:

A increasing	B decreasing
C constant and equal to mgh	D zero

Solution: What the potential energy loses the kinetic energy gains, so the total stays at its starting value.**MCQ 4.8.4** For a body falling through a height h against a frictional force f :

A $mgh = \frac{1}{2}mv^2$	B $mgh = \frac{1}{2}mv^2 + fh$
C $mgh = fh$	D $mgh + fh = \frac{1}{2}mv^2$

Solution: The loss in potential energy is the gain in kinetic energy plus the work done against friction.**MCQ 4.8.5** The relation "loss in P.E. = gain in K.E." is true only when:

A the body is heavy	B the height is large
C friction is neglected	D the body is at rest

Solution: Any friction takes a share of the potential energy, so the gain in kinetic energy falls short by that much.**MCQ 4.8.6** A ball is dropped from a height of 20 m. Neglecting air resistance, it strikes the ground with a speed of about:

A 10 m s^{-1}	B 14 m s^{-1}
C 19.8 m s^{-1}	D 39.2 m s^{-1}

Solution: $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 20} = 19.8 \text{ m s}^{-1}$ **Chapter Exercise****SQ 4.1** Why is electrical power required when an elevator is descending? Why should there be a limit on the number of passengers in this case?**Need for Electrical Power**

The motor does work against friction and controls the downward motion.

Limit on Passengers

More passengers increase the weight and hence the work done by the motor.
The load must stay within the safe power rating of the motor.

SQ 4.2 A body is being raised to a height H from the surface of the Earth. What is the sign of work done by both the body and the Earth? Justify.

Positive Work

Work done on the body is positive because the applied force and the displacement are in the same direction.

$$W = FH \cos 0^\circ = +mgH$$

Negative Work

Work done by the Earth is negative because the gravitational force is opposite to the displacement.

$$W = FH \cos 180^\circ = -mgH$$

SQ 4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during the fall? Justify.

No, its mechanical energy is not conserved.

Justification

Air resistance is a non-conservative force.
A part of P.E. is used in doing work against air friction.
The remaining P.E. is converted into K.E.

$$\text{Loss in P.E.} = \text{Gain in K.E.} + \text{Work done against friction}$$

SQ 4.4 Calculate the power of a crane in kilowatts which lifts a mass of 1000 kg to a height of 100 m in 20 s.

Given Data

$$m = 1000 \text{ kg}$$

$$h = 100 \text{ m}$$

$$t = 20 \text{ s}$$

Work Done

$$W = mgh = (1000)(9.8)(100) = 9.8 \times 10^5 \text{ J}$$

Power

$$P = \frac{W}{t} = \frac{9.8 \times 10^5}{20} = 49 \text{ kW}$$

Result $P = 49 \text{ kW}$

SQ 4.5 A trolley of mass 1500 kg carrying 500 kg of sand moves uniformly at 40 km h^{-1} on a frictionless track. Sand leaks vertically at 0.05 kg s^{-1} . What is the trolley speed after all the sand has leaked out?

Given

$$m_i = 1500 + 500 = 2000 \text{ kg}, \quad v_i = 40 \text{ km h}^{-1}, \quad m_f = 1500 \text{ kg}$$

By Conservation of Momentum

$$m_i v_i = m_f v_f$$

$$(2000)(40) = (1500) v_f$$

$$v_f = \frac{80000}{1500} = 53.33 \text{ km h}^{-1}$$

SQ 4.6 Give the absolute and gravitational units of work in M.K.S. and C.G.S. systems.

MKS

Absolute unit: joule (N m).

Gravitational unit: kilogram-force metre (kgf m).

CGS

Absolute unit: erg (dyn cm).

Gravitational unit: gram-force centimetre (gf cm).

Relation $1 \text{ J} = 10^7 \text{ erg}$

SQ 4.7 A body dropped from a height H reaches the ground with speed $1.2\sqrt{gH}$. Calculate the work done by air friction.

Apply the energy relation

$$\text{Loss in P.E.} = \text{Gain in K.E.} + \text{work against friction}$$

Therefore

$$mgH = \frac{1}{2}m(1.2\sqrt{gH})^2 + W_f$$

$$W_f = 0.28mgH$$

SQ 4.8 A bicycle has kinetic energy of 150 J. What kinetic energy would it have if it had (i) the same mass but double the speed, and (ii) three times the mass and one-half the speed?

$$\text{Relation} \quad K.E. = \frac{1}{2}mv^2 = 150 \text{ J}$$

(i) Same mass and double speed

$$K.E.' = \frac{1}{2}m(2v)^2 = 4 \left(\frac{1}{2}mv^2 \right)$$

$$K.E.' = 4(150) = 600 \text{ J}$$

(ii) Three times mass and one-half speed

$$K.E.' = \frac{1}{2}(3m)\left(\frac{v}{2}\right)^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$$

$$K.E.' = \frac{3}{4}(150) = 112.5 \text{ J}$$

SQ 4.9 What is the effect on the kinetic energy of a body of mass m moving with velocity v when its momentum becomes double? Justify.

Momentum and Velocity

The momentum is the product of the mass and the velocity, so for a fixed mass a double momentum means a double velocity.

$$p = mv$$

New Kinetic Energy

$$K.E.' = \frac{1}{2}m(2v)^2 = 4 \left(\frac{1}{2}mv^2 \right)$$

Result

The kinetic energy becomes four times its original value.

SQ 4.10 Does the International Space Station have gravitational potential energy, kinetic energy, or both? Explain.

The International Space Station possesses both K.E. and gravitational P.E.

It has K.E. due to its orbital motion.

It has P.E. due to its position in the gravitational field of the Earth.

$$K.E. = \frac{1}{2}mv^2$$

$$P.E. = -\frac{GMm}{r}$$

CRQ 4.1 When is a force said to be conservative? Give two conditions.

A force is conservative if its work between two points is independent of the path followed.

Condition 1

Work done between two points is path independent.

Condition 2

Work done in a closed path is zero.

CRQ 4.2 A light body and a heavy body have the same linear momentum. Which one has greater kinetic energy?

Relation

$$K.E. = \frac{p^2}{2m}$$

For the same momentum, K.E. is inversely proportional to mass.

$$\frac{p^2}{2} = \text{constant}$$

$$K.E. \propto \frac{1}{m}$$

Therefore, the light body has greater K.E.

CRQ 4.3 A motorcycle moves with constant speed on a horizontal track. Is any work being done on it if no net force acts on it?

No work is done in this case.

By Work Energy Theorem

$$W = \text{change in K.E.}$$

The speed is constant, so the K.E. of the motorcycle remains constant.

Therefore, the change in K.E. is zero and no net work is done.

CRQ 4.4 A force acts on a ball moving at 14 m s^{-1} and reduces its speed to 6 m s^{-1} . Has the force done positive or negative work? Explain.

Apply the work-energy theorem

$$W = (K.E.)_f - (K.E.)_i$$

$$W = \frac{1}{2}m(6^2 - 14^2) = -80m \text{ J}$$

The work is negative because the K.E. of the ball decreases.

CRQ 4.5 A slow-moving truck can have more kinetic energy than a fast-moving car. How is this possible?

K.E. depends upon mass and the square of velocity.

$$K.E. = \frac{1}{2}mv^2$$

The mass of a truck is much greater than that of a car. Therefore, a slow-moving truck may possess greater K.E.

CRQ 4.6 Why is work done against friction non-conservative in nature? Explain briefly.

Work done against friction depends upon the length of the path followed.

Work done in a closed path is not zero.

Therefore, friction is non-conservative.

CRQ 4.7 Does wind contain kinetic energy? Explain.

Yes, wind contains kinetic energy.

Wind is air in motion, and air has mass and velocity.

$$K.E. = \frac{1}{2}mv^2$$

MCQ 4.1 A 1 kg mass has a potential energy of 1 J relative to the ground at a height of:

A	0.102 m	B	1 m
C	3.8 m	D	32 m

$$\text{Solution: } h = \frac{P.E.}{mg} = \frac{1}{1 \times 9.8} = 0.102 \text{ m}$$

MCQ 4.2 An iron sphere of mass 30 kg and an aluminium sphere of mass 10.5 kg have the same diameter and are dropped together from a cliff. At 10 m above the ground they have identical:

A	accelerations	B	momentums
C	potential energies	D	kinetic energies

Solution: Both fall freely, so both have the acceleration g ; the unequal masses make the momenta and the energies differ.

MCQ 4.3 A body at rest may have:

A	speed	B	velocity
C	momentum	D	energy

Solution: At rest the speed, velocity and momentum are all zero, but the body can still possess potential energy.

MCQ 4.4 A child on a swing rises from 0.5 m at the lowest point to 1.5 m at the highest point. The maximum speed of the child is about:

A	1.5 m s^{-1}	B	4.4 m s^{-1}
C	9.8 m s^{-1}	D	depends upon the child's mass

Solution: The child falls through $h = 1.5 - 0.5 = 1 \text{ m}$, so $v = \sqrt{2gh} = 4.4 \text{ m s}^{-1}$

MCQ 4.5 A ball is thrown vertically upward and falls back to the ground. The conservative force in this case is:

A	air resistance	B	gravity
C	friction between ball and air	D	contact force with hand

Solution: The work done by gravity is path independent and zero in a closed path.

MCQ 4.6 By the work-energy principle in linear motion, the work done on a body equals the:

A	change in K.E.	B	change in P.E.
C	zero	D	sum of K.E. and P.E.

Solution: The work done by the net force equals the change in kinetic energy of the body.

MCQ 4.7 The power of a lamp is 6 W. The energy it gives out in 2 min is:

A	12 J	B	20 J
C	3 J	D	720 J

Solution: $E = Pt = 6 \times 120 = 720 \text{ J}$

MCQ 4.8 A dry battery delivers 3000 J of energy to a 2 W motor before it is exhausted. The battery runs for:

A	1500 min	B	100 min
C	50 min	D	25 min

$$\text{Solution: } t = \frac{E}{P} = \frac{3000}{2} = 1500 \text{ s} = 25 \text{ min}$$

MCQ 4.9 The kinetic energy gained by a mass m after moving a fixed distance from rest under a constant force is proportional to:

A	$\sqrt{m}$	B	$1/\sqrt{m}$
C	m	D	independent of m

Solution: Here $K.E. = Fd$, which has no mass term, so it is independent of m

MCQ 4.10 A body moves 10 m in a straight line under a 5 N force. If the work done is 25 J, the angle between the force and the motion is:

A	0°	B	30°
C	60°	D	90°

$$\text{Solution: } \cos \theta = \frac{W}{Fd} = \frac{25}{50} = 0.5, \text{ so } \theta = 60^\circ$$

Numerical 4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1} . If each bullet has a mass of 40 g, then find the power developed by the gun. (Ans: 980 W)

Given Data

$$n = 6 \text{ bullets per minute}$$

$$v = 700 \text{ m s}^{-1}$$

$$m = 40 \text{ g} = 0.04 \text{ kg}$$

$$t = 60 \text{ s}$$

To Find Power developed $P = ?$

Formula

The gun supplies the kinetic energy of every bullet it fires.

$$K.E. = \frac{1}{2}mv^2$$

$$P = \frac{n \times K.E.}{t}$$

Solution

Kinetic Energy of One Bullet

$$K.E. = \frac{1}{2} \times 0.04 \times (700)^2 = 9800 \text{ J}$$

Power

$$P = \frac{6 \times 9800}{60} = \frac{58800}{60} = 980 \text{ W}$$

Result $P = 980 \text{ W}$

Numerical 4.2 A family uses 10 kW of power. Direct solar energy is incident on a horizontal surface at an average rate of 300 W per square metre. If 75% of this energy can be converted into useful electrical energy, how large an area is needed to supply 10 kW? (Ans: 44.44 m²)

Given Data

$$P = 10 \text{ kW} = 10000 \text{ W}$$

$$I = 300 \text{ W m}^{-2}$$

$$\eta = 75\% = 0.75$$

To Find Area required $A = ?$

Formula

Only the useful fraction of the incident power is available.

$$P = \eta IA$$

$$A = \frac{P}{\eta I}$$

Solution

Putting the values in the formula

$$A = \frac{10000}{0.75 \times 300} = \frac{10000}{225} = 44.44 \text{ m}^2$$

Result $A = 44.44 \text{ m}^2$

Numerical 4.3 The mass of the Earth is $6.0 \times 10^{24} \text{ kg}$ and the mass of the Sun is $1.99 \times 10^{30} \text{ kg}$. The Sun is 160 million km away from the Earth. Find the value of the gravitational potential energy of the Earth. (Ans: $-4.97 \times 10^{33} \text{ J}$)

Given Data

$$m = 6.0 \times 10^{24} \text{ kg}$$

$$M = 1.99 \times 10^{30} \text{ kg}$$

$$r = 160 \times 10^9 = 1.6 \times 10^{11} \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

To Find

Gravitational potential energy $U = ?$

Formula

The absolute gravitational potential energy at a distance r is negative.

$$U = -\frac{GMm}{r}$$

Solution

Putting the values in the formula

$$U = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 6.0 \times 10^{24}}{1.6 \times 10^{11}} = -\frac{7.96 \times 10^{44}}{1.6 \times 10^{11}} = -4.97 \times 10^{33} \text{ J}$$

Result $U = -4.97 \times 10^{33} \text{ J}$

Numerical 4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1} . What is the frictional force acting on it? (Ans: 26 N)

Given Data

$$w = 98 \text{ N}$$

$$h = 10 \text{ m}$$

$$v = 12 \text{ m s}^{-1}$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find

Frictional force $f = ?$

Formula

The loss in potential energy is the gain in kinetic energy plus the work done against friction.

$$wh = \frac{1}{2}mv^2 + fh$$

$$m = \frac{w}{g} = 10 \text{ kg}$$

Solution

Putting the values in the formula

$$98 \times 10 = \frac{1}{2} \times 10 \times (12)^2 + f \times 10$$

$$980 = 720 + 10f$$

$$f = \frac{980 - 720}{10} = \frac{260}{10} = 26 \text{ N}$$

Result $f = 26 \text{ N}$

Numerical 4.5 A 75 W fan is used for 8 hours daily for 30 days. Find (i) the energy consumed in electrical units and (ii) the electricity bill if one unit costs Rs. 22.5. [Ans: (i) 18 units (ii) Rs. 405]

Given Data

$$P = 75 \text{ W} = 0.075 \text{ kW}$$

$$t = 8 \text{ h per day} \times 30 \text{ days} = 240 \text{ h}$$

$$\text{Rate} = \text{Rs. } 22.5 \text{ per unit}$$

To Find

Energy consumed $E = ?$ and the bill $= ?$

Formula

One electrical unit is one kilowatt-hour.

$$E = Pt$$

$$\text{Bill} = E \times \text{Rate}$$

Solution

Energy Consumed

$$E = 0.075 \text{ kW} \times 240 \text{ h} = 18 \text{ kWh}$$

Electricity Bill

$$\text{Bill} = 18 \times 22.5 = \text{Rs. } 405$$

Result

$$E = 18 \text{ units}$$

$$\text{Bill} = \text{Rs. } 405$$

Numerical 4.6 An object of mass 2 kg thrown up from the ground reaches a height of 5 m and falls back to the Earth, neglecting air resistance. Calculate (i) the work done by gravity as the object reaches the height of 5 m, (ii) the work done by gravity as it comes back to the Earth and (iii) the total work done by gravity, mentioning the physical significance of the result. [Ans: (i) -98 J (ii) +98 J (iii) 0]

Given Data

$$m = 2 \text{ kg}$$

$$h = 5 \text{ m}$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find

Work done by gravity going up, coming down and in the whole trip.

$$\text{Formula } W = \vec{F} \cdot \vec{d} = mgh \cos \theta$$

Solution

Going Up

The weight is opposite to the displacement, so the angle is 180°

$$W_{up} = -2 \times 9.8 \times 5 = -98 \text{ J}$$

Coming Down

The weight is along the displacement, so the angle is zero.

$$W_{down} = +2 \times 9.8 \times 5 = +98 \text{ J}$$

Total Work Done $W = -98 + 98 = 0$

Result $W = 0$

Significance

The work done in a closed path in a conservative field is zero, which is why gravity is a conservative force.

Numerical 4.7 An electrical motor of one horsepower is used to run a water pump. The water pump takes 15 minutes to fill a tank of 400 litres at a height of 10 m. Take 1 hp = 746 W. Find (a) the actual input work done by the electric motor to fill the tank and (b) the actual output work done. [Ans: (a) 671.4 kJ (b) 39.2 kJ]

Given Data

$$P = 1 \text{ hp} = 746 \text{ W}$$

$$t = 15 \text{ min} = 900 \text{ s}$$

$$V = 400 \text{ litre} \Rightarrow m = 400 \text{ kg}$$

$$h = 10 \text{ m}$$

To Find

Input work $W_{in} = ?$ and output work $W_{out} = ?$

Formula $W_{in} = Pt$

$$W_{out} = mgh$$

Solution

Input Work

$$W_{in} = 746 \times 900 = 671400 \text{ J}$$

Output Work

One litre of water has a mass of one kilogram.

$$W_{out} = 400 \times 9.8 \times 10 = 39200 \text{ J}$$

Result

$$W_{in} = 671.4 \text{ kJ}$$

$$W_{out} = 39.2 \text{ kJ}$$

Numerical 4.8 A passenger has just arrived at the airport and is dragging his suitcase to the luggage check-in desk. He pulls the strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of the work done by him on the suitcase. (Ans: 7 kJ)

Given Data $F = 200 \text{ N}$

$$\theta = 45^\circ$$

$$d = 50 \text{ m}$$

To Find Work done $W = ?$

Formula

Only the component of the force along the displacement does work.

$$W = Fd \cos \theta$$

Solution

Putting the values in the formula

$$W = 200 \times 50 \times \cos 45^\circ = 10000 \times 0.707 = 7071 \text{ J}$$

Result $W \approx 7 \text{ kJ}$

Numerical 4.9 A 1200 kg car is running at a speed of 40 km h^{-1} . How much power will be expended by it to accelerate at 2 m s^{-2} ? (Ans: 26.67 kW)

Given Data

$$m = 1200 \text{ kg}$$

$$v = 40 \text{ km h}^{-1} = 11.11 \text{ m s}^{-1}$$

$$a = 2 \text{ m s}^{-2}$$

To Find Power expended $P = ?$

Formula

Power is the scalar product of the force and the velocity.

$$F = ma$$

$$P = Fv = mav$$

Solution

Putting the values in the formula

$$P = 1200 \times 2 \times 11.11 = 26664 \text{ W}$$

Result $P = 26.67 \text{ kW}$

Numerical 4.10 A 200 g apple is lifted to 10 m and then dropped. What is its velocity when it hits the ground? Assume that 75% of the work done in lifting the apple is transferred to kinetic energy by the time it hits the ground. (Ans: 12.1 m s^{-1})

Given Data

$$m = 200 \text{ g} = 0.2 \text{ kg}$$

$$h = 10 \text{ m}$$

$$\eta = 75\% = 0.75$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find

Velocity on hitting the ground $v = ?$

Formula

Three quarters of the work done in lifting the apple reappears as kinetic energy.

$$W = mgh$$

$$\frac{1}{2}mv^2 = \eta W$$

$$v = \sqrt{2\eta gh}$$

Solution

Work Done in Lifting $W = 0.2 \times 9.8 \times 10 = 19.6 \text{ J}$

Kinetic Energy on Landing

$$\frac{1}{2} \times 0.2 \times v^2 = 0.75 \times 19.6 = 14.7 \text{ J}$$

$$v^2 = \frac{2 \times 14.7}{0.2} = 147 \text{ m}^2 \text{ s}^{-2}$$

Result $v = 12.1 \text{ m s}^{-1}$

Self Test

Seventeen questions across the whole chapter. Work through them without the notes, then check the answers keyed at the foot of the page.

Q 1 Work is done on a body only when the applied force produces a:

- | | | | |
|---|-----------------------|---|--------------|
| A | change in mass | B | displacement |
| C | change in temperature | D | sound |

Q 2 A body is carried horizontally on the head of a coolie. The work done by him against the weight of the load is:

- | | | | |
|---|-------|---|------------------|
| A | mgd | B | $-mgd$ |
| C | zero | D | $\frac{1}{2}mgd$ |

Q 3 A force of 12 N moves a body 3 m along its own direction. The work done is:

- | | | | |
|---|------|---|------|
| A | 4 J | B | 15 J |
| C | 36 J | D | 9 J |

Q 4 The work done by a variable force is obtained from the graph plotted between:

- | | | | |
|---|-------------|---|-------------------------|
| A | F and t | B | $F \cos \theta$ and d |
| C | v and t | D | P and t |

Q 5 Which of the following pairs are both conservative forces?

- | | | | |
|---|----------------------------|---|--|
| A | friction and tension | B | gravitational and elastic spring force |
| C | air resistance and gravity | D | normal force and rocket propulsion |

Q 6 The work done by a conservative force around a closed path ACBA is:

- | | | | |
|---|--------|---|--------|
| A | mgh | B | $-mgh$ |
| C | $2mgh$ | D | zero |

Q 7 A machine does 600 J of work in 2 s. Its power is:

- | | | | |
|---|--------|---|-------|
| A | 1200 W | B | 300 W |
| C | 30 W | D | 60 W |

Q 8 Power expressed as the scalar product of two quantities is:

- | | | | |
|---|-------------------------|---|-------------------------|
| A | $\vec{F} \cdot \vec{d}$ | B | $\vec{F} \cdot \vec{a}$ |
| C | $\vec{F} \cdot \vec{v}$ | D | $\vec{v} \cdot \vec{d}$ |

Q 9 The kinetic energy of a body is halved when its speed is:

- | | | | |
|---|-----------------------|---|--------------------------|
| A | halved | B | doubled |
| C | divided by $\sqrt{2}$ | D | multiplied by $\sqrt{2}$ |

Q 10 The absolute potential energy of a body becomes zero at:

- | | | | |
|---|---|---|-------------------------------------|
| A | the surface of the Earth | B | the centre of the Earth |
| C | a height equal to the radius of the Earth | D | an infinite distance from the Earth |

Q 11 The escape velocity from the surface of the Earth is independent of the:

- | | | | |
|---|----------------------------|---|-----------------------------|
| A | mass of the Earth | B | radius of the Earth |
| C | mass of the projected body | D | value of g at the surface |

Q 12 The work-energy theorem relates the work done by a net force to the change in:

- | | | | |
|---|----------------|---|------------------|
| A | kinetic energy | B | potential energy |
| C | momentum | D | power |

Q 13 A body of mass 4 kg moving at 3 m s^{-1} is brought to rest. The work done on it is:

- | | | | |
|---|-------|---|-------|
| A | +18 J | B | -18 J |
| C | +36 J | D | -36 J |

Q 14 For a body falling freely from rest, the quantity that stays the same throughout the fall is the:

- | | | | |
|---|----------------|---|---|
| A | kinetic energy | B | potential energy |
| C | velocity | D | sum of the kinetic and the potential energy |

Q 15 A body falls through a height h against a frictional force f . Its kinetic energy on reaching the ground is:

- | | | | |
|---|------------|---|------------|
| A | mgh | B | fh |
| C | $mgh - fh$ | D | $mgh + fh$ |

Q 16 One kilowatt-hour of electrical energy is equal to:

- | | | | |
|---|--------|---|--------|
| A | 1000 J | B | 3600 J |
| C | 3.6 MJ | D | 36 MJ |

Q 17 A body is projected from the surface of the Earth with a speed of 12 km s^{-1} . It will:

- | | | | |
|---|--------------------------|---|--|
| A | fall back to the ground | B | escape from the gravitational field of the Earth |
| C | go into a circular orbit | D | stay at rest above the ground |

Answers 1 (b) 2 (c) 3 (c) 4 (b) 5 (b) 6 (d) 7 (b) 8 (c) 9 (c) 10 (d) 11 (c) 12 (a) 13 (b) 14 (d) 15 (c) 16 (c) 17 (b)