TOPIC 1 Work Done by a Constant Force

SQ 4.1.1 What two things does the term work involve in physics?

Answer

In physics the term work involves two things, force and displacement.

SQ 4.1.2 Define work done by a constant force.

Definition

When a constant force F displaces an object through a displacement d in the direction of the force, the work done is the product of the magnitude of the force and the magnitude of the displacement.

Formula

$W = Fd$

SQ 4.1.3 Why is no work done when we push against a wall?

Reason

If the displacement is zero, no work is done even if a large force is applied. Pushing on a wall may tire the muscles, but the work done is zero.

SQ 4.1.4 Write the expression for work when the force makes an angle with the displacement.

Explanation

The work done is the product of the component of force along the direction of displacement and the magnitude of the displacement.

Formula

$W = (F\cos\theta)d = Fd\cos\theta$

SQ 4.1.5 Express work done as a scalar product.

Formula

$W = \mathbf{F}\cdot\mathbf{d}$

This shows that work is a scalar quantity.

SQ 4.1.6 Write the SI unit of work.

SI Unit

The unit of work is joule (J), and

$1\,\mathrm{J} = 1\,\mathrm{N\,m}.$

SQ 4.1.7 When is the work done said to be positive?

Answer

If $\theta < 90^\circ$, work is done and it is said to be positive work.

SQ 4.1.8 When is no work done by a force?

Answer

If

$\theta = 90^\circ$

then

$\cos\theta = 0$

and hence no work is done.

SQ 4.1.9 When is the work done said to be negative?

Answer

If $\theta > 90^\circ$, the work done is said to be negative.

SQ 4.1.10 How is work represented on a force-displacement graph?

Answer

The distance is plotted along the x-axis and the force along the y-axis. Since a constant force does not vary, the graph is a horizontal straight line.

Result

The area under the force-displacement curve represents the work done by the constant force.

SQ 4.1.11 What is plotted on the graph when the force is not in the direction of displacement?

Answer

In that case the graph is plotted between $F\cos\theta$ and d.

SQ 4.1.12 Is any work being done when a motorcycle runs with constant speed on a horizontal track?

Answer

The driving force does work against friction, but no work is done against gravity, because the weight of the motorcycle is perpendicular to the horizontal displacement and

$\theta = 90^\circ$

TOPIC 2 Work Done by a Variable Force

SQ 4.2.1 Why is a variable force considered separately?

Reason

In many cases the force does not remain constant during the process of doing work, so the simple formula

$W = Fd$

cannot be applied directly.

SQ 4.2.2 Give two examples where the force varies.

Examples

As a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre.

The force exerted by a spring increases with the amount of stretch.

SQ 4.2.3 How is the work done by a variable force calculated?

Method

The path is divided into n short intervals of displacement, and during each small interval the force is supposed to be approximately constant.

Formula

$$W_{total} = \sum_{i=1}^{n} F_i\cos\theta_i\,\Delta d_i$$

SQ 4.2.4 Write the exact expression for the work done by a variable force.

Formula

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i$$

SQ 4.2.5 Why does subdividing the path into more intervals give a more accurate result?

Reason

If the distance is subdivided into a large number of intervals so that each Δd becomes very small, the assumption of constant force in each interval becomes better. Letting each Δd approach zero gives an exact result.

SQ 4.2.6 How is the work done by a variable force found from a graph?

Answer

The work done by a variable force in moving a particle between two points is equal to the area under the $F\cos\theta$ versus d curve between those two points.

SQ 4.2.7 What does the shaded rectangle on an F cos θ versus d graph represent?

Answer

The ith shaded rectangle has an area $F_i \cos\theta_i \, \Delta d$, which is the work done during the ith interval.

SQ 4.2.8 A force is 5 N from d = 0 to 4 m and falls to zero at d = 6 m. Find the work done from 0 to 4 m.

Solution

The work done is represented by the area of the rectangle.

Calculation

$W = 4\,\text{m} \times 5\,\text{N}$

Result

$W = 20\,\text{J}$

SQ 4.2.9 For the same force, find the work done from d = 4 m to d = 6 m.

Solution

The work done is represented by the area of the triangle.

Calculation

$W = \frac{1}{2} \times 2\,\text{m} \times 5\,\text{N}$

Result

$W = 5\,\text{J}$

SQ 4.2.10 Find the total work done by the above force as the object moves from d = 0 to d = 6 m.

Solution

The total work is the area of the rectangular section plus the area of the triangular section.

Calculation

$W = 20\,\text{J} + 5\,\text{J}$

Result

$W = 25\,\text{J}$

TOPIC 3 Conservative and Non-Conservative Forces

SQ 4.3.1 What is a gravitational field?

Definition

The space around the Earth in which its gravitational force acts on a body is called the gravitational field.

SQ 4.3.2 When is the work done by the gravitational force positive and when is it negative?

Positive Work

When the displacement is in the direction of the gravitational force.

Negative Work

When the displacement is against the gravitational force.

SQ 4.3.3 Show that the work done by gravity along path ADB is independent of the path.

Derivation

The work done along AD is zero, because the weight mg is perpendicular to this path.
The work done along DB is $-mgh$, because the direction of mg is opposite to the displacement, so
$\theta = 180^\circ$

Result

$W_{ADB} = 0 + (-mgh) = -mgh$

SQ 4.3.4 What is the work done by gravity along a curved path from A to B?

Explanation

The curved path is imagined to be broken down into a series of horizontal and vertical steps. No work is done along the horizontal steps because mg is perpendicular to them.

Result

Work is done only along the vertical displacements, and the net work is again $-mgh$.

SQ 4.3.5 State the conclusion drawn about the work done by gravitational force.

Conclusion

Work done by the gravitational force is independent of the path followed.

SQ 4.3.6 Define a conservative force.

Definition

If the work done by a force in moving an object between two points is independent of the path followed, or the work done in a closed path is zero, the force is called a conservative force.

SQ 4.3.7 Give examples of conservative forces.

Examples

The gravitational force is a conservative force. Other examples are the electrostatic force and the elastic spring force.

SQ 4.3.8 Define a non-conservative force.

Definition

A force is non-conservative if the work done by it in moving an object between two points, or in a closed path, depends on the path of motion.

SQ 4.3.9 Why is kinetic friction a non-conservative force?

Reason

When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path.

Result

A greater amount of work is done over a longer path, so the work depends on the choice of path.

SQ 4.3.10 What is the total work done by a non-conservative force in a closed path?

Answer

The total work done by a non-conservative force in a closed path is not zero.

SQ 4.3.11 Give examples of non-conservative forces.

Examples

The kinetic frictional force, air resistance, tension in a string, normal force and the propulsion force of a rocket.

SQ 4.3.12 Differentiate between conservative and non-conservative forces.

Conservative Force	Non-Conservative Force
The work done is independent of the path followed.	The work done depends on the path of motion.
The work done in a closed path is zero.	The work done in a closed path is not zero.
Gravitational and elastic spring forces are examples.	Friction and air resistance are examples.

SQ 4.3.13 Show that the gravitational field is conservative in nature.

Proof

The work done by gravity in moving a body from A to B is $-mgh$ along the straight path ADB, along the path ACB and along any curved path.

Result

Since the work done is the same for every path and is zero for a closed path such as $ACBA$, the gravitational field is conservative in nature.

TOPIC 4 Power

SQ 4.4.1 Why is the concept of power needed?

Reason

In the definition of work it is not clear whether the same amount of work is done in one second or in one hour. The rate at which work is done is often of interest in practical applications.

SQ 4.4.2 Define power.

Definition

Power is the measure of the rate at which work is being done.

SQ 4.4.3 Write the formula for average power.

Formula

$$P_{av} = \frac{\Delta W}{\Delta t}$$

SQ 4.4.4 Write the formula for instantaneous power.

Formula

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Here ΔW is the work done in the short interval of time Δt.

SQ 4.4.5 Express power as a scalar product of force and velocity.

Derivation

Since

$$\Delta W = \mathbf{F} \cdot \Delta \mathbf{d}$$

therefore

$$P = \mathbf{F} \cdot \frac{\Delta \mathbf{d}}{\Delta t}$$

Formula

$$P = \mathbf{F} \cdot \mathbf{v}$$

SQ 4.4.6 Define the SI unit of power.

SI Unit

The SI unit of power is the watt, defined as one joule of work done in one second.

SQ 4.4.7 What is the commercial unit of electrical energy?

Answer

The commercial unit of electrical energy is the kilowatt-hour.

SQ 4.4.8 Define one kilowatt-hour and find its value in joules.

Definition

One kilowatt-hour is the work done in one hour by an agency whose power is one kilowatt.

Calculation

$$1\,\mathrm{kWh} = 1000\,\mathrm{W} \times 3600\,\mathrm{s}$$

Result

$$1\,\mathrm{kWh} = 3.6 \times 10^6\,\mathrm{J} = 3.6\,\mathrm{MJ}$$

SQ 4.4.9 A 70 kg man runs up stairs of vertical height 4.5 m in 4.0 s. Find his power output.

Solution

The work done is

$$W = mgh$$

and the power is

$$P = \frac{mgh}{t}$$

Calculation

$$P = \frac{70 \times 9.8 \times 4.5}{4.0}$$

Result

$$P = 7.7 \times 10^2\,\mathrm{W}$$

TOPIC 5 Energy

SQ 4.5.1 Define energy and name its two basic forms.

Definition

Energy of a body is its capacity to do work.

Forms

There are two basic forms of energy, kinetic energy and potential energy.

SQ 4.5.2 Define kinetic energy and potential energy.

Kinetic Energy

Kinetic energy is the energy possessed by a body due to its motion.

Potential Energy

Potential energy is the energy possessed by a body due to its changed position.

SQ 4.5.3 Which two energies are kinds of mechanical energy?

Answer

The kinetic energy and the potential energy are both kinds of mechanical energy.

SQ 4.5.4 Derive the formula for kinetic energy.

Derivation

A moving car does work fd against friction before stopping, and its acceleration is

$$a = -\frac{f}{m}$$

Using

$$2aS = v_f^2 - v_i^2$$

with

$$v_i = v$$

and

$$v_f = 0$$

gives

$$2\left(-\frac{f}{m}\right)d = -v^2.$$

Result

$$KE = \frac{1}{2}mv^2$$

SQ 4.5.5 Why is the unit of kinetic energy the joule?

Reason

Kinetic energy is equal to the work which the body is capable of doing, so its unit must be that of work, that is the joule.

SQ 4.5.6 A 18620 N car moving at 16 m s^{-1} stops in 80 m. Find its mass.

Solution

$$m = \frac{w}{g} = \frac{18620}{9.8}$$

Result

$$m = 1900\,\mathrm{kg}$$

SQ 4.5.7 A 1900 kg car moving at 16 m s^{-1} is brought to rest in 80 m. Find the average force of friction.

Solution

The kinetic energy of the car equals the work done before stopping, so

$$\frac{1}{2}mv^2 = fd$$

Calculation

$$f = \frac{1900 \times (16)^2}{2 \times 80}$$

Result

$$f = 3040\,\mathrm{N}$$

SQ 4.5.8 Why does a body possess potential energy?

Answer

Potential energy is possessed by a body because of its position in a force field, such as a gravitational field, or because of its constrained state.

SQ 4.5.9 What is elastic potential energy?

Definition

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called elastic potential energy.

SQ 4.5.10 Define absolute potential energy.

Definition

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity, where the force of gravity becomes zero.

SQ 4.5.11 When is the relation P.E. = mgh valid?

Answer

The relation mgh is true only near the surface of the Earth, where the gravitational force is nearly constant.

SQ 4.5.12 Why can P.E. = mgh not be used for large distances?

Reason

If the body is displaced through a large distance in space, the gravitational force will not remain constant, since it varies inversely as the square of the distance.

SQ 4.5.13 How is the difficulty of a varying gravitational force overcome?

Method

The distance between the two points is divided into small steps, each of length Δr, so that the value of the force remains constant for each small step. The total work is then found by adding the work done during all these steps.

SQ 4.5.14 Write the gravitational force at the centre of a small step.

Formula

$$F = \frac{GMm}{r^2}$$

Here m is the mass of the object, M is the mass of the Earth and G is the gravitational constant.

SQ 4.5.15 Why is the term (Δr)^2 neglected in the derivation of absolute potential energy?

Reason

Since $(\Delta r)^2 \ll r_1 r_2$, therefore $(\Delta r)^2$ can be neglected as compared to $r_1 r_2$, which gives

$$r^2 = r_1 r_2$$

SQ 4.5.16 Write the work done in displacing a body from point 1 to point N.

Formula

$$W_{1\to N} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

SQ 4.5.17 Derive the expression for absolute potential energy at a distance r.

Derivation

If the point N is situated at an infinite distance, then

$$\frac{1}{r_N} = 0$$

Result

$$U = -\frac{GMm}{r}$$

SQ 4.5.18 Why does the potential energy increase as r increases?

Reason

When r increases,

$$U = -\frac{GMm}{r}$$

becomes less negative, that is U increases. It means that when we raise a body above the surface of the Earth, its potential energy increases.

SQ 4.5.19 Write the absolute potential energy on the surface of the Earth.

Formula

$$U_g = -\frac{GMm}{R}$$

Here R is the radius of the Earth.

SQ 4.5.20 What does the negative sign in the expression for absolute potential energy show?

Answer

The negative sign shows that the Earth's gravitational field for a mass m is attractive.

SQ 4.5.21 How much work must be done to raise a body to an infinite distance?

Answer

We will have to do work on it equal to $\dfrac{GMm}{R}$, so that its potential energy becomes zero.

TOPIC 6 Escape Velocity

SQ 4.6.1 Why does an object projected upward come back to the ground?

Reason

The object comes back to the ground after rising to a certain height because of the force of gravity acting downward.

SQ 4.6.2 What happens as the initial velocity of a projected object is increased?

Answer

With increased initial velocity the object rises to a greater height before coming back. If we go on increasing the initial velocity, a stage comes when it will not return to the ground and will escape from the influence of gravity.

SQ 4.6.3 Define escape velocity.

Definition

The initial velocity of an object with which it goes out of the Earth's gravitational field is known as escape velocity.

SQ 4.6.4 To what does escape velocity correspond?

Answer

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of the Earth.

SQ 4.6.5 Write the increase in potential energy in lifting a body to infinity.

Formula

$$\Delta U = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

SQ 4.6.6 Derive the expression for escape velocity.

Derivation

The body escapes if its initial kinetic energy equals the increase in potential energy, so

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}.$$

Result

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

SQ 4.6.7 Write escape velocity in terms of g and R.

Derivation

Since

$$g = \frac{GM}{R^2}$$

therefore

$$gR = \frac{GM}{R}$$

Result

$$v_{esc} = \sqrt{2gR}$$

SQ 4.6.8 What is the value of escape velocity from the Earth?

Value

The value of escape velocity from the Earth comes out to be approximately $11\,\mathrm{km\,s^{-1}}$.

SQ 4.6.9 Write the escape speeds from the Moon and Mercury.

Values

The escape speed from the Moon is $2.4\,\mathrm{km\,s^{-1}}$ and from Mercury is $4.3\,\mathrm{km\,s^{-1}}$.

SQ 4.6.10 Write the escape speeds from Jupiter and Saturn.

Values

The escape speed from Jupiter is $60.0\,\mathrm{km\,s^{-1}}$ and from Saturn is $37.0\,\mathrm{km\,s^{-1}}$.

SQ 4.6.11 Does the escape velocity depend on the mass of the escaping body?

Answer

No. Since

$$v_{esc} = \sqrt{2gR}$$

the mass m of the body cancels out, so the escape velocity is independent of the mass of the escaping body.

TOPIC 7 Work-Energy Theorem

SQ 4.7.1 **What happens whenever work is done on a body?**

Answer
Whenever work is done on a body, it increases its energy.

SQ 4.7.2 **Derive the work-energy theorem.**

Derivation
From
$2ad = v_f^2 - v_i^2$
we get
$d = \dfrac{v_f^2 - v_i^2}{2a}$
and from the second law
$F = ma$
Multiplying gives
$Fd = ma \times \dfrac{v_f^2 - v_i^2}{2a}.$

Result
$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

SQ 4.7.3 **State the work-energy theorem.**

Statement
The change in kinetic energy of an object is equal to the work done on it by the net force.

Formula
$W = (KE)_f - (KE)_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

SQ 4.7.4 **By what other name is the work-energy theorem known?**

Answer
The work-energy theorem is also known as the work-energy principle.

SQ 4.7.5 **Is the work-energy theorem valid for any direction of the force?**

Answer
Yes. The work-energy theorem is applicable for any direction of the force relative to the displacement.

SQ 4.7.6 **What happens when an object with kinetic energy pushes another object?**

Answer
An object with kinetic energy can perform work if it is allowed to push or pull on another object. In this case the work will be taken as negative and the kinetic energy of the object will decrease.

SQ 4.7.7 **Does the work-energy theorem remain valid for a variable force?**

Answer
Yes. The theorem remains valid even if the force may vary from point to point.

SQ 4.7.8 **A 90 kg motorcycle and rider coast down a 24° slope against a friction force of 100 N. Find the net force.**

Solution
The net force is
$F = mg\sin 24^\circ - f$

Calculation
$F = (90 \times 9.8 \times 0.4) - 100$

Result
$F = 252.8\,\mathrm{N}$

SQ 4.7.9 **For the above motorcycle, find the work done over 72 m downhill.**

Solution
$W = Fd = 252.8 \times 72$

Result
$W = 18201.6\,\mathrm{J}$

SQ 4.7.10 **A 90 kg motorcycle starts at 3.2 m s⁻¹ and gains 18201 J of work. Find its final speed.**

Solution
Using the work-energy theorem
$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Calculation
$\frac{1}{2}(90)v_f^2 = 18201 + \frac{1}{2}(90)(3.2)^2$

Result
$v_f = 20.4\,\mathrm{m\,s^{-1}}$

SQ 4.7.11 **A force acts on a ball moving at 14 m s⁻¹ and brings its speed to 6 m s⁻¹. Has work been done?**

Answer
Yes, work has been done on the ball.

Reason
By the work-energy theorem the work done equals the change in kinetic energy. Since the speed decreases, the kinetic energy decreases and the work done on the ball is negative.

SQ 4.7.12 **Why is the normal force balanced in the motorcycle example?**

Answer
The normal force F_n is balanced by the component of weight $mg\cos 24^\circ$ perpendicular to the slope, so it does not contribute to the net force along the slope.

TOPIC 8 Interconversion of Potential and Kinetic Energy

SQ 4.8.1 **What are the potential and kinetic energies of a body at rest at a height h?**

Answer
At position A the body has
$PE = mgh$
and
$KE = 0$

SQ 4.8.2 **Write the potential energy of a falling body after it has fallen a distance x.**

Answer
The height from the ground is $(h - x)$.

Formula
$PE = mg(h - x)$

SQ 4.8.3 **Find the velocity of a falling body after it has fallen a distance x.**

Solution
Using
$v_f^2 = v_i^2 + 2gS$
with
$v_i = 0$
and
$S = x$
Result
$v_B = \sqrt{2gx}$

SQ 4.8.4 **Write the kinetic energy of a falling body after it has fallen a distance x.**

Solution
$KE = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2gx)$
Result
$KE = mgx$

SQ 4.8.5 **Show that the total energy at an intermediate point B is mgh.**

Solution
$\text{Total energy} = PE + KE = mg(h-x) + mgx$
Result
$\text{Total energy} = mgh$

SQ 4.8.6 **What are the potential and kinetic energies just before the body strikes the Earth?**

Answer
At position C the potential energy is zero and the kinetic energy is $\frac{1}{2}mv_C^2$, where
$v_C^2 = 2gh$
Result
$KE = mgh$

SQ 4.8.7 **What is concluded at point C about kinetic energy?**

Conclusion
At point C the kinetic energy is equal to the original value of the potential energy of the body.

SQ 4.8.8 **Why does the kinetic energy of a falling body increase?**

Reason
When a body falls its velocity increases, that is the body is accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy.

SQ 4.8.9 **Why does the potential energy of a falling body decrease?**

Reason
As the body falls, its height decreases and hence its potential energy also decreases.

SQ 4.8.10 **State the relation between loss in P.E. and gain in K.E.**

Statement
Loss in potential energy is equal to the gain in kinetic energy.

Formula
$mg(h_1 - h_2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$
This result is true only when the frictional force is not considered.

SQ 4.8.11 **What happens to the potential energy when friction is present during downward motion?**

Answer
A part of the potential energy is used in doing work against friction, equal to fh. The remaining potential energy $mgh - fh$ is converted into kinetic energy.

SQ 4.8.12 **Write the energy equation for downward motion in the presence of friction.**

Formula
$mgh = \frac{1}{2}mv^2 + fh$
Statement
Loss in potential energy equals gain in kinetic energy plus work done against friction.

SQ 4.8.13 **State the energy relation for upward motion in the presence of friction.**

Statement
Loss of kinetic energy is equal to the gain in potential energy plus the work done against friction.

SQ 4.8.14 **Will the total mechanical energy of a body falling in air be conserved?**

Answer
No, it will not be conserved.
Reason
Air resistance is a non-conservative force, so part of the mechanical energy is used in doing work against friction and is converted into heat.

SQ 4.8.15 **A body at rest may have which forms of energy?**

Answer
A body at rest has no kinetic energy, but it may still have potential energy because of its position in a force field or because of its constrained state.