

Topic 7.1 Waves

Wave

A wave is a regular disturbance or variation that carries energy, which spreads out from the source.

Energy Transfer

Energy is transferred from the Sun to the Earth in the form of light waves, called electromagnetic waves. These waves can even travel through vacuum.

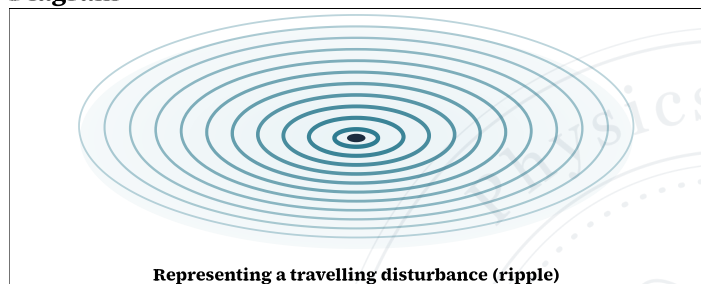
In a Medium

Energy is transferred due to the regular and repeated disturbances that travel through the medium, making its particles move up and down or back and forth.

Example

A stone thrown into a pond produces a disturbance, a ripple, that travels through the water.

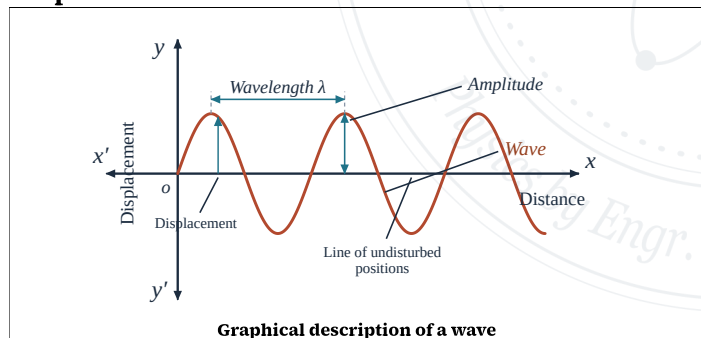
- The water particles only move up and down about their mean positions.
- The repeating pattern that spreads out is the wave.

Diagram**Displacement**

The displacement of a particle of a wave is its distance in a specified direction from its rest or equilibrium position.

Waveform

When the displacement is plotted along the y-axis and the time in the direction of energy travel along the x-axis, the graph obtained is called the waveform.

Graph**Characteristics of a Wave****Amplitude (A)**

The maximum displacement of the particles of the medium from their equilibrium position.

Frequency (f)

The number of oscillations or cycles per second.

Wavelength (λ)

The distance between two consecutive similar points on the wave that are in phase.

Period (T)

The time taken by the wave to complete one oscillation. It is the reciprocal of the frequency.

$$T = \frac{1}{f}$$

Speed (v)

The speed at which the wave travels. A crest moves one wavelength λ in one period T

$$v = \frac{\lambda}{T}$$

Since the reciprocal of the period is the frequency

$$v = f\lambda$$

Phase (θ)

The relative position of a point on the wave at a given time.

Types of Waves**Mechanical Waves**

They require a physical medium — solid, liquid or gas — to propagate. Water waves, sound waves and seismic waves are mechanical waves.

Electromagnetic Waves

They do not require a medium and can travel through vacuum. Radio waves, microwaves, infrared, visible light, ultraviolet, X-rays and gamma rays are electromagnetic waves.

Quantum Waves

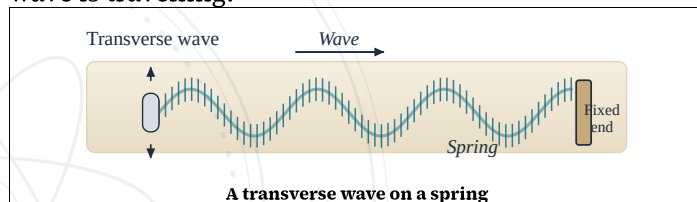
They are associated with particles such as electrons and photons. Matter waves, also called de Broglie waves, and photon waves are quantum waves.

Surface Waves

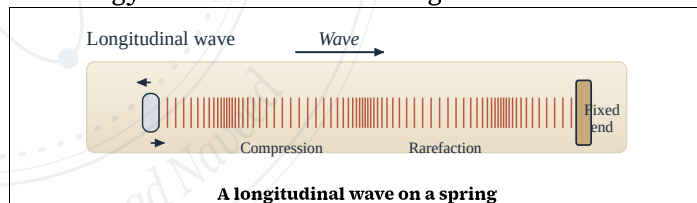
They propagate along surfaces or interfaces between two media, such as wind-driven ocean surface waves and seismic surface waves.

Transverse and Longitudinal Waves**Transverse Wave**

A wave in which the vibrations of the particles are at right angles to the direction in which the energy of the wave is travelling.

**Longitudinal Wave**

A wave in which the direction of the vibration of the particles is along or parallel to the direction in which the energy of the wave is travelling.

**Short Questions**

SQ 7.1.1 A wave transfers energy without transferring matter. Explain this with the example of a ripple on a pond.

Energy Travels, Matter Does Not

The stone gives the water a disturbance, and it is that disturbance which spreads outwards across the pond.

Motion of the Particles

Each water particle only moves up and down about its mean position and returns to it. No water is carried from the centre to the edge of the pond.

SQ 7.1.2 Using the definitions of wavelength and time period, obtain the wave speed equation.

Distance Covered in One Period

A crest moves forward one whole wavelength in one time period.

$$v = \frac{\lambda}{T}$$

Frequency

The frequency is the reciprocal of the time period.

$$f = \frac{1}{T}$$

Result

$$v = f\lambda$$

SQ 7.1.3 What are quantum waves? Give two examples.	
Quantum Waves Waves that are associated with particles such as <u>electrons and photons</u> . Examples Matter waves, also called de Broglie waves, such as the electron waves in an atom. Photon waves, that is, the waves of light quanta.	
SQ 7.1.4 Light from the Sun reaches the Earth but sound does not. Why?	
Different Kinds of Wave Light is an electromagnetic wave and sound is a mechanical wave. Reason An electromagnetic wave needs <u>no medium</u> and crosses the vacuum of space, while a mechanical wave needs particles of a medium to pass the disturbance on, and space between the Sun and the Earth has almost none.	
SQ 7.1.5 A wave of frequency 500 Hz travels with a speed of 340 m s⁻¹. Calculate its wavelength.	
Formula $v = f\lambda$ $\lambda = \frac{v}{f}$ Calculation $\lambda = \frac{340}{500} = 0.68 \text{ m}$	
SQ 7.1.6 What is meant by the phase of a wave, and when are two points said to be in phase?	
Phase The phase is the <u>relative position</u> of a point on the wave at a given time. In Phase Two points are in phase when they are at the same stage of their motion, that is, they have the same displacement moving in the same direction. Two consecutive crests are in phase, and the distance between them is one wavelength.	
SQ 7.1.7 Name the four types of waves classified by their nature, giving one example of each.	
Mechanical Waves Sound waves in air. Electromagnetic Waves Visible light. Quantum Waves The electron waves in an atom, that is, de Broglie waves. Surface Waves Wind-driven ocean surface waves.	
MCQ 7.1.1 What a wave carries from one place to another is:	
A matter	B the medium itself
C energy	D mass
Solution: A wave is a disturbance that transports energy; the particles of the medium only oscillate about their mean positions.	
MCQ 7.1.2 The maximum displacement of the particles of the medium from the equilibrium position is the:	
A wavelength	B period
C phase	D amplitude
Solution: That maximum displacement is the definition of the amplitude.	
MCQ 7.1.3 The distance between two consecutive points of a wave that are in phase is its:	
A amplitude	B period
C wavelength	D frequency
Solution: Two successive crests are in phase, and the distance between them is one wavelength.	

MCQ 7.1.4 The reciprocal of the time period of a wave is its:	
A speed	B frequency
C wavelength	D amplitude
Solution: One cycle takes a time T , so the number of cycles per second is $1/T$.	
MCQ 7.1.5 A wave of wavelength 2 m has a frequency of 170 Hz. Its speed is:	
A 85 m s ⁻¹	B 172 m s ⁻¹
C 680 m s ⁻¹	D 340 m s⁻¹
Solution: The speed is the product of frequency and wavelength, 170×2 .	
MCQ 7.1.6 Waves which need no medium to travel are:	
A mechanical waves	B surface waves
C electromagnetic waves	D water waves
Solution: Electromagnetic waves travel through vacuum, which is how sunlight reaches the Earth.	
MCQ 7.1.7 De Broglie waves are an example of:	
A surface waves	B mechanical waves
C quantum waves	D electromagnetic waves
Solution: Matter waves associated with particles such as electrons are quantum waves.	
MCQ 7.1.8 In a longitudinal wave the particles of the medium vibrate:	
A at right angles to the flow of energy	B in circles
C along the direction of the flow of energy	D not at all
Solution: The vibration is parallel to the direction in which the energy travels, giving compressions and rarefactions.	
MCQ 7.1.9 Waves that travel along the boundary between two media are called:	
A quantum waves	B surface waves
C electromagnetic waves	D matter waves
Solution: Ocean surface waves and seismic surface waves propagate along an interface.	

Topic 7.2 Principle of Superposition of Waves

LQ 1 starts here — State and explain the principle of superposition of waves. Apply this principle to elaborate the working of noise cancelling headphones.

Statement

If a particle of the medium is simultaneously acted upon by two waves, then the resultant displacement of the particle is the algebraic sum of their individual displacements.

Meaning

The displacements of the individual waves are added together to form a new wave pattern.

Waves in Phase

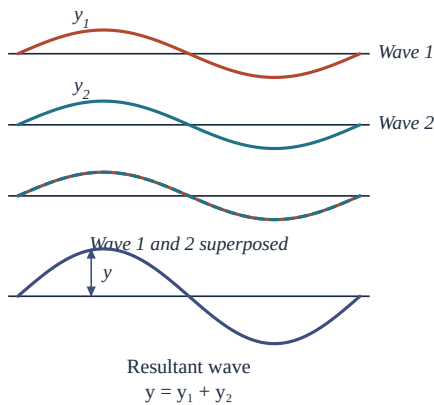
If two waves which overlap each other have the same phase, their resultant displacement is

$$y = y_1 + y_2$$

Here y_1 and y_2 are the amplitudes of the two waves and y is the resultant amplitude. If the two amplitudes are equal, then

$$y = 2y_1$$

Diagram



Superposition of two waves of the same frequency which are exactly in phase

Waves Out of Phase

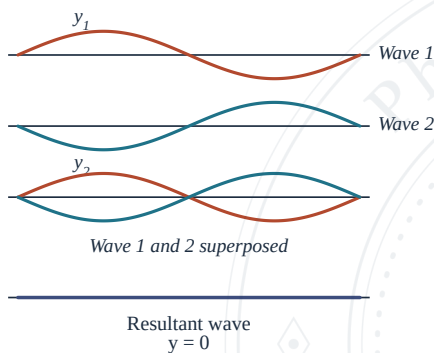
If two waves which cross each other have opposite phase, their resultant displacement is

$$y = y_1 - y_2$$

If the two amplitudes are equal, then the resultant displacement is zero.

$$y = 0$$

Diagram



Superposition of two waves of the same frequency which are exactly out of phase

General Statement

If a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual waves is y_1 and y_2 up to y_n , then the resultant displacement is the algebraic sum of all the displacements.

$$y = y_1 + y_2 + \dots + y_n$$

As a Wave Function

$$y(x, t) = y_1(x, t) + y_2(x, t) + \dots + y_n(x, t)$$

Here $y(x, t)$ is the resultant wave and the terms on the right are the individual waves. The wave function represents the wave displacement at a given point x and time t , and describes the shape of the wave and its evolution over time.

Limitation

The principle of superposition applies to linear waves, that is, to waves of small amplitude.

Phenomena It Leads To

- Two waves having the same frequency and travelling in the same direction give interference.
- Two waves of slightly different frequencies travelling in the same direction give beats.
- Two waves of equal frequency travelling in opposite directions give stationary waves.

Applications of the Principle of Superposition

Noise-Cancelling Headphones

By applying the principle of superposition of waves, noise-cancelling headphones eliminate unwanted noise and give a more peaceful listening experience.

Working

- The headphones contain one or more microphones that capture the ambient noise, such as background chatter or engine rumble.
- The microphone sends the sound signals to an amplifier and a processing unit in the headphones.
- The processing unit generates an anti-noise signal, which is the exact opposite of the ambient noise in both amplitude and phase.
- The anti-noise signal is played through the headphones along with the desired audio.
- When the anti-noise signal meets the ambient noise, the two waves cancel each other out and a much quieter sound is heard.

Note

Real headphones use complex algorithms and several microphones, but the basic principle at work is superposition.

LQ 1 ends here

Short Questions

SQ 7.2.1 Two waves of amplitudes 3 cm and 5 cm act at the same point. Find the greatest and the least resultant amplitude.

In Phase

The displacements add.

$$y = 5 + 3 = 8 \text{ cm}$$

Out of Phase

The displacements subtract.

$$y = 5 - 3 = 2 \text{ cm}$$

SQ 7.2.2 To which waves does the principle of superposition apply?

Linear Waves

It applies to linear waves, that is, to waves of small amplitude.

Reason

Only then do the two disturbances pass through each other unchanged, so that the resultant is the simple algebraic sum of the two displacements.

SQ 7.2.3 Name the three phenomena that follow from the principle of superposition and give the condition for each.

Interference

Two waves of the same frequency travelling in the same direction.

Beats

Two waves of slightly different frequencies travelling in the same direction.

Stationary Waves

Two waves of equal frequency travelling in opposite directions.

SQ 7.2.4 What does the wave function $y(x, t)$ represent?

Wave Function

It is the wave displacement at a given point x and at a given time t .

What It Tells

It describes the shape of the wave and how that shape evolves over time.

SQ 7.2.5 Two identical waves arrive at a point exactly out of phase. What is the resultant displacement there, and what has become of the energy?

Zero Displacement

The two displacements are equal and opposite, so the resultant is zero.

$$y = y_1 - y_2 = 0$$

The Energy

No energy is destroyed. It is redistributed to the points where the two waves arrive in phase, and those points receive more than they would from either wave alone.

MCQ 7.2.1 Two waves of amplitude 4 cm each meet exactly in phase. The resultant amplitude is:

- A zero
C 4 cm
B 8 cm
D 16 cm

Solution: Equal displacements in the same direction add, giving $2y_1$.

MCQ 7.2.2 Two waves of amplitude 4 cm each meet exactly out of phase. The resultant amplitude is:

- A zero
C 4 cm
B 8 cm
D 2 cm

Solution: Equal and opposite displacements cancel, so the resultant is zero.

MCQ 7.2.3 The principle of superposition applies to:

- A linear waves of small amplitude
C sound waves only
B waves of any amplitude
D stationary waves only

Solution: The simple algebraic sum holds only while the waves are linear, that is, of small amplitude.

MCQ 7.2.4 Two waves of the same frequency travelling in the same direction produce:

- A beats
C interference
B stationary waves
D diffraction

Solution: Same frequency and same direction is the condition for interference.

MCQ 7.2.5 Two waves of slightly different frequencies travelling in the same direction produce:

- A beats
C stationary waves
B interference
D refraction

Solution: The small difference in frequency makes the loudness rise and fall, which is a beat.

MCQ 7.2.6 Two waves of equal frequency travelling in opposite directions produce:

- A stationary waves
C interference
B beats
D diffraction

Solution: Opposite directions is the condition for a standing wave pattern of nodes and antinodes.

MCQ 7.2.7 The anti-noise signal of a noise-cancelling headphone is the exact opposite of the ambient noise in:

- A amplitude only
C speed only
B frequency only
D amplitude and phase

Solution: Only a signal matched in amplitude and opposite in phase can cancel the noise on superposing with it.

MCQ 7.2.8 In the superposition equation, $y(x, t)$ represents the:

- A speed of the wave
C total energy of the wave
B wave displacement at a given point and time
D phase constant

Solution: The wave function gives the displacement at the point x at the time t .

Topic 7.3 Interference and Its Types

Interference

Superposition of two waves having the same frequency and travelling in the same direction results in a phenomenon called interference.

Experimental Setup

Two loudspeakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency produced by an audio generator.

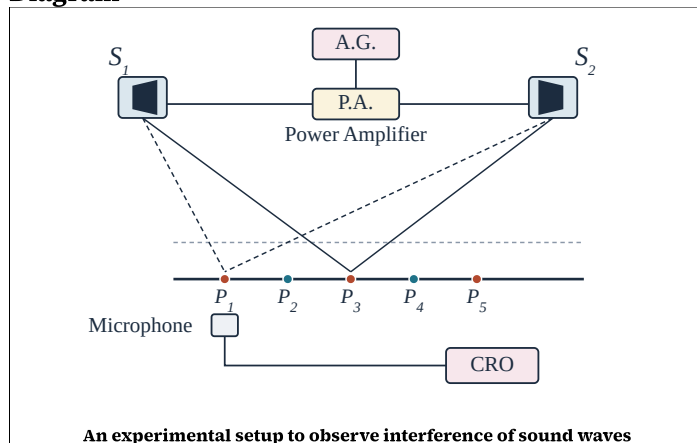
Coherent Sources

Since the two speakers are driven from the same generator, they vibrate in phase. Such sources of waves are called coherent sources.

Detector

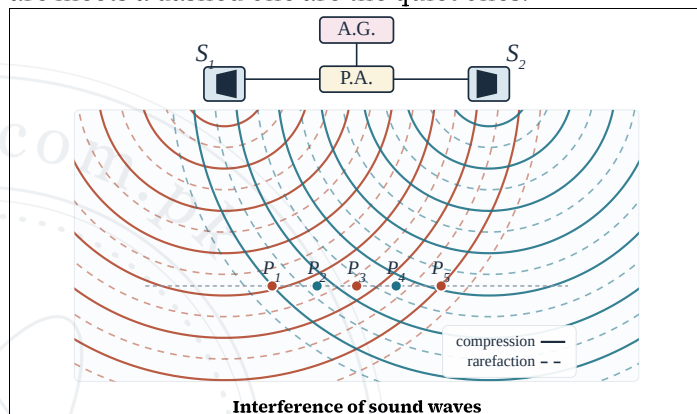
A microphone attached to a sensitive cathode ray oscilloscope acts as a detector of sound waves. The CRO displays the input signal as a waveform on its screen, and the microphone is placed at various points in front of the loudspeakers turn by turn.

Diagram



Wavefronts

The solid arcs are the compressions and the dashed arcs the rarefactions, so the points where two solid arcs meet are the loud points and the points where a solid arc meets a dashed one are the quiet ones.



Constructive Interference

At the points P_1 , P_3 and P_5 a compression meets a compression and a rarefaction meets a rarefaction. The displacements of the two waves are added up at these points and a large resultant displacement is seen on the CRO screen.

Path Difference

The path difference ΔS between the waves at the point P_3 is

$$\Delta S = S_2P_3 - S_1P_3 = 0$$

At the points P_1 and P_5 the path difference is one wavelength. Here λ is the distance between any two successive solid or dashed lines.

Condition

Whenever the path difference is an integral multiple of the wavelength, the two waves are added up.

$$\Delta S = n\lambda$$

Here n is a whole number.

$$n = 0, \pm 1, \pm 2, \pm 3, \dots$$

Destructive Interference

At the points P_2 and P_4 a compression meets a rarefaction, so the two cancel each other's effect and the resultant displacement becomes zero on the CRO screen.

Path Difference

The path difference between the waves at the point P_2 is

$$\Delta S = S_2P_2 - S_1P_2 = \frac{\lambda}{2}$$

At the point P_4 the path difference is $3\lambda/2$.

Condition

At points where the displacements of the two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength.

$$\Delta S = (2n + 1) \frac{\lambda}{2}$$

Here n is a whole number.

$$n = 0, \pm 1, \pm 2, \pm 3, \dots$$

Book Examples

Example 7.3.1 Two speakers are arranged 3.0 m apart and emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to, and 4.0 m from, the line connecting the two speakers. A tone of maximum loudness is heard when the microphone is at the centre of the line and when it is directly opposite each speaker. Calculate the speed of sound.

Given Data

$$S_1 S_2 = 3.0 \text{ m}$$

$$f = 344 \text{ Hz}$$

$$S_1 P = 4.0 \text{ m}$$

To Find Speed of sound $v = ?$

Formula

For maximum loudness the path difference must be an integral multiple of the wavelength.

$$\Delta S = n\lambda$$

$$v = f\lambda$$

Solution

At the middle point O the two paths are equal, so the path difference is zero and constructive interference takes place there. At the next point P_1 of constructive interference the path difference is one wavelength.

$$\lambda = S_2 P_1 - S_1 P_1$$

The distance $S_2 P_1$ is found from the right-angled triangle $S_1 S_2 P_1$ by Pythagoras theorem.

$$S_2 P_1 = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5 \text{ m}$$

So the wavelength is

$$\lambda = 5 - 4 = 1 \text{ m}$$

Putting this in the wave speed equation

$$v = 344 \times 1 = 344 \text{ m s}^{-1}$$

Result $v = 344 \text{ m s}^{-1}$

Example 7.3.2 The wavelength of a signal from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz, and with what speed do the radio waves travel?

Given Data

$$\lambda_1 = 1500 \text{ m}$$

$$f_1 = 200 \text{ kHz} = 2.0 \times 10^5 \text{ Hz}$$

$$f_2 = 1000 \text{ kHz} = 1 \times 10^6 \text{ Hz}$$

To Find

Speed of the waves $v = ?$ and wavelength $\lambda_2 = ?$

Formula $v = f\lambda$

Solution

The speed of both signals is the same, so it is found from the first transmitter.

$$v = 2.0 \times 10^5 \times 1500 = 3.0 \times 10^8 \text{ m s}^{-1}$$

For the second transmitter

$$\lambda_2 = \frac{v}{f_2} = \frac{3.0 \times 10^8}{1 \times 10^6} = 3 \times 10^2 \text{ m}$$

Result

$$v = 3 \times 10^8 \text{ m s}^{-1}$$

$$\lambda_2 = 3 \times 10^2 \text{ m}$$

Short Questions

SQ 7.3.1 What is the part played by the cathode ray oscilloscope in the interference experiment with sound?

Detector

The microphone converts the sound at a point into an electrical signal, and the CRO displays that signal as a

waveform on its screen.

What It Shows

A large trace where a compression meets a compression, and a flat trace where a compression meets a rarefaction.

SQ 7.3.2 At a point the path difference between two sound waves from coherent sources is 2.5λ . What is heard there?

Odd Multiple of Half a Wavelength

$$\Delta S = 2.5\lambda = 5 \left(\frac{\lambda}{2} \right)$$

Answer

The path difference is an odd multiple of half the wavelength, so the interference is destructive and the point is quiet.

SQ 7.3.3 Is energy destroyed at a point of destructive interference?

No

Energy is not destroyed anywhere in an interference pattern.

Where It Goes

The energy missing from the quiet points appears at the loud points, so that the total energy carried by the two waves is unchanged.

SQ 7.3.4 Two loudspeakers driven by separate generators of the same frequency give no steady interference pattern. Why?

Not Coherent

Two separate generators do not hold a constant phase difference; it drifts from moment to moment.

Consequence

The positions of the loud and the quiet points move about too quickly to be observed, so no steady pattern is seen.

SQ 7.3.5 Two speakers in phase send sound to a point 6.00 m from one and 6.75 m from the other, where the sound is a minimum. Find the longest wavelength and the lowest frequency that fit, taking the speed of sound as 340 m s^{-1} .

Path Difference

$$\Delta S = 6.75 - 6.00 = 0.75 \text{ m}$$

Condition

The longest wavelength comes from the first minimum, where the path difference is just half a wavelength.

$$\Delta S = \frac{\lambda}{2}$$

$$\lambda = 2 \times 0.75 = 1.5 \text{ m}$$

Frequency

$$f = \frac{v}{\lambda} = \frac{340}{1.5} = 226.7 \text{ Hz}$$

SQ 7.3.6 Why is the interference of sound easier to demonstrate in the laboratory than the interference of light?

Wavelength

The wavelength of audible sound is of the order of a metre, while that of light is less than a micrometre.

Consequence

The loud and quiet points of sound are far enough apart to be found by walking a microphone across the room, whereas the bright and dark fringes of light are so close together that a special arrangement is needed to separate them.

MCQ 7.3.1 In the loudspeaker experiment the microphone is connected to a:

A cathode ray oscilloscope

B voltmeter

C galvanometer

D spectrometer

Solution: The CRO displays the signal picked up by the microphone as a waveform.

MCQ 7.3.2 Constructive interference occurs where the path difference is:

- A an odd multiple of $\lambda/2$ B an integral multiple of λ
 C $\lambda/4$ D $\lambda/3$

Solution: A whole number of wavelengths brings the two waves back into phase.

MCQ 7.3.3 Destructive interference occurs where the path difference is:

- A $n\lambda$ B $2n\lambda$
 C an odd integral multiple of $\lambda/2$ D $n\lambda/4$

Solution: An odd number of half wavelengths makes a compression arrive with a rarefaction.

MCQ 7.3.4 At a point where a compression meets a rarefaction the CRO shows:

- A zero displacement B a large displacement
 C a doubled frequency D a shift of phase only

Solution: The two displacements cancel, so the trace is flat.

MCQ 7.3.5 The path difference at the mid-point of the line joining two coherent sources is:

- A $\lambda/2$ B λ
 C zero D $3\lambda/2$

Solution: The mid-point is equidistant from both sources, so the two paths are equal.

MCQ 7.3.6 Two speakers 3.0 m apart face a detector line 4.0 m away. At the point directly opposite one speaker the path difference is:

- A zero B 0.5 m
 C 5.0 m D 1.0 m

Solution: The far path is $\sqrt{3^2 + 4^2} = 5$ m and the near path is 4 m, so the difference is 1 m.

MCQ 7.3.7 If the path difference at a point is 3λ , the interference there is:

- A constructive B destructive
 C partial D absent

Solution: Three whole wavelengths is an integral multiple of λ .

MCQ 7.3.8 A path difference of 2.5λ produces:

- A constructive interference B no interference at all
 C destructive interference D beats

Solution: 2.5λ is five half wavelengths, an odd multiple of $\lambda/2$.

Topic 7.4 Stationary Waves and Their Formation

LQ 2 starts here – What are standing waves? Illustrate a detailed experiment that demonstrates the standing waves using stretched strings.

Stationary Waves

Stationary waves, also known as standing waves, are the waves that oscillate in a fixed position, without moving or propagating.

Formation

They are formed by the superposition of two waves of the same frequency and amplitude travelling in opposite directions.

Pattern

The resulting wave pattern remains stationary, with nodes, the points of zero amplitude, and antinodes, the points of maximum amplitude, at fixed positions. Waves on a string and sound waves in a pipe are examples.

Name

The term standing wave describes that the wave pattern remains fixed in space, oscillating between positive and negative values without moving forward or backward.

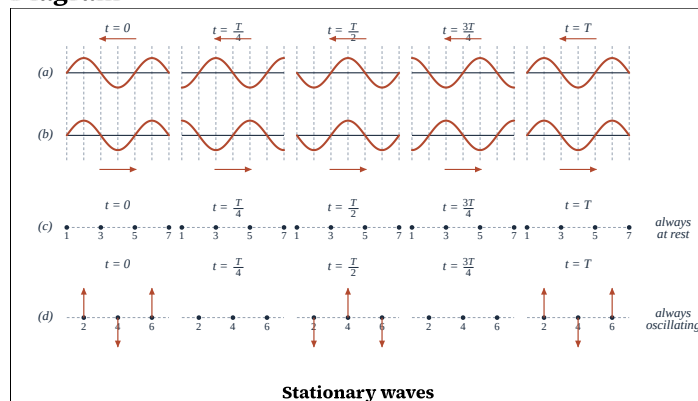
Superposition on a String

Consider two waves moving along a string in opposite directions. Their profiles are drawn at five instants of one time period T

$$t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$$

The points 1 to 7 marked on the string are $\lambda/4$ apart, and the resultant displacement of each is found by applying the principle of superposition.

Diagram



Nodes

The resultant displacement of the points 1, 3, 5 and 7 is always zero at every instant. Such points of the medium are known as nodes.

Distance Between Nodes

$$\text{node to node} = \frac{\lambda}{2}$$

Antinodes

The points 2, 4 and 6 move with an amplitude which is the sum of the amplitudes of the component waves. These points are known as antinodes. They are situated midway between the nodes and are $\lambda/2$ apart.

Distance Between a Node and an Antinode

$$\text{node to antinode} = \frac{\lambda}{4}$$

Stationary Wave

Such a pattern of nodes and antinodes is known as a stationary or standing wave.

Energy in a Stationary Wave

Energy in a wave transfers because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points.

Where the Energy Stays

The energy remains standing in the medium between the nodes, although it alternates between potential and kinetic forms at the antinodes.

- When the antinodes are all at their extreme displacements, the energy stored is wholly potential.
- When they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

Short Questions

SQ 7.4.1 In a stationary wave, how far apart are two consecutive nodes, and how far is a node from the next antinode?

Node to Node node to node = $\frac{\lambda}{2}$

Node to Antinode

The antinodes lie midway between the nodes.

$$\text{node to antinode} = \frac{\lambda}{4}$$

SQ 7.4.2 Why can energy not flow past a node of a stationary wave?

Nodes are at Rest

Energy in a wave is carried by the motion of the particles of the medium, and a node never moves.

Consequence

With no moving particle to pass the energy on, the energy stays standing in the medium between one node and the next.

SQ 7.4.3 At an antinode, when is the energy wholly potential and when is it wholly kinetic?

Wholly Potential

When the antinodes are all at their extreme displacements and momentarily at rest.

Wholly Kinetic

When they are simultaneously passing through their equilibrium positions at maximum speed.

SQ 7.4.4 The nodes of a stationary wave on a string are 0.25 m apart. Find the wavelength of the waves that formed it.

Formula

The distance between consecutive nodes is half a wavelength.

$$\frac{\lambda}{2} = 0.25 \text{ m}$$

Calculation $\lambda = 2 \times 0.25 = 0.5 \text{ m}$

SQ 7.4.5 Differentiate between a progressive wave and a stationary wave.

Progressive Wave	Stationary Wave
The wave profile <u>moves forward</u> .	The pattern stays <u>fixed in space</u> .
Every particle has the same amplitude.	The amplitude varies from zero at a node to a maximum at an antinode.
Energy is carried from one place to another.	Energy remains standing between the nodes.
It is formed by a single wave.	It is formed by two waves travelling in opposite directions.

MCQ 7.4.1 A stationary wave is formed by two waves of the same frequency and amplitude travelling:

- A in opposite directions B in the same direction
C at right angles D in random directions

Solution: Only two oppositely directed waves give a pattern that stands still.

MCQ 7.4.2 The distance between two consecutive nodes of a stationary wave is:

- A λ B $\lambda/4$
C 2λ D $\lambda/2$

Solution: The nodes recur every half wavelength.

MCQ 7.4.3 The distance between a node and the next antinode is:

- A $\lambda/2$ B λ
C $\lambda/8$ D $\lambda/4$

Solution: The antinode lies midway between two nodes that are $\lambda/2$ apart.

MCQ 7.4.4 In a stationary wave the energy:

- A flows past the nodes B remains standing between the nodes
C is destroyed at the nodes D flows only at the nodes

Solution: A node never moves, so it can pass no energy along.

MCQ 7.4.5 When all the antinodes are momentarily at their extreme displacements, the energy of a stationary wave is:

- A wholly kinetic B half kinetic and half potential
C wholly potential D zero

Solution: At the extreme displacement the particles are at rest, so no kinetic energy is left.

MCQ 7.4.6 A stationary wave has its nodes 0.3 m apart. The wavelength of the component waves is:

- A 0.6 m B 0.15 m
C 0.3 m D 1.2 m

Solution: The node spacing is half a wavelength, so $\lambda = 2 \times 0.3$.

MCQ 7.4.7 In the book's figure of a stationary wave, the marked points 1 to 7 are separated by:

- A $\lambda/4$ B $\lambda/2$
C λ D $\lambda/8$

Solution: The seven points are a quarter of a wavelength apart, so they fall alternately on nodes and antinodes.

Topic 7.5 Stationary Waves on a Stretched String

The Arrangement

Consider a string of length ℓ which is kept stretched by clamping its ends, so that the tension in the string is F .

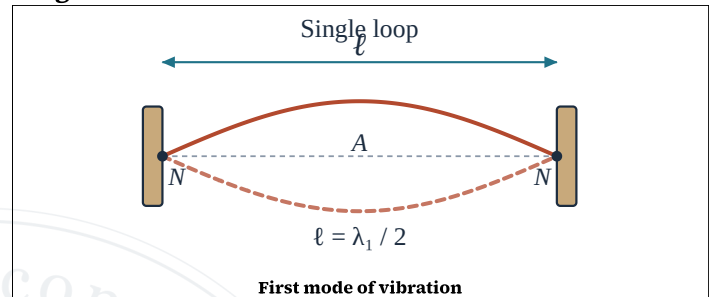
String Plucked at Its Middle Point

Two transverse waves originate from the middle point, one moving towards the left end and the other towards the right end. When these waves reach the clamped ends they are reflected back, and the incident and reflected waves give rise to stationary waves.

Mode of Vibration

As the two ends of the string are clamped, no motion takes place there, so nodes are formed at the two ends with one antinode in between. The string seems to vibrate in one loop.

Diagram



Wavelength

The distance between two consecutive nodes is half the wavelength, so the length of the string is

$$\ell = \frac{\lambda_1}{2}$$
$$\lambda_1 = 2\ell$$

Speed of the Waves

The speed of the waves in the string depends upon the tension F and the mass per unit length m of the string. It is independent of the point from where the string is plucked.

$$v = \sqrt{\frac{F}{m}}$$

Fundamental Frequency

$$f_1 = \frac{v}{\lambda_1}$$

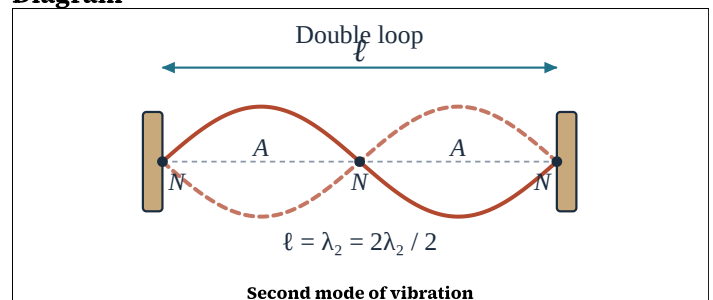
$$f_1 = \frac{1}{2\ell} \sqrt{\frac{F}{m}}$$

In this first mode of vibration, waves of the frequency f_1 only will be set up in the string.

String Plucked at Quarter Length

If the same string is plucked at one quarter of its length, stationary waves are again set up, but now the string vibrates in two loops. This configuration develops because the string was plucked from the position of an antinode.

Diagram



Wavelength

The distance between two consecutive nodes is half the wavelength, so the length of the string is equal to the whole wavelength.

$$\ell = \lambda_2$$

The wavelength in this case is half of that in the first case, while the speed depends only on the tension and the mass per unit length and so remains the same.

Second Harmonic

$$f_2 = \frac{v}{\lambda_2}$$
$$f_2 = \frac{v}{\ell} = 2 \left(\frac{v}{2\ell} \right) = 2f_1$$

Result

When the string vibrates in two loops, its frequency becomes double that of the one-loop mode.

String Plucked at an Arbitrary Point

Let the string resonate in n loops, with $n + 1$ nodes and n antinodes. The frequency of the stationary waves set up on the string is

$$f_n = n \left(\frac{v}{2\ell} \right) = nf_1$$

and the corresponding wavelength is

$$\lambda_n = \frac{2\ell}{n}$$

Here n is a whole number.

$$n = 1, 2, 3, \dots$$

Effect on Frequency and Wavelength

As the string vibrates in more than one loop, its frequency goes on increasing and the wavelength gets correspondingly shorter. The product of the frequency and the wavelength is always equal to the speed of the waves.

Harmonic Series

The stationary waves have a discrete set of frequencies which is known as the harmonic series.

$$f_1, 2f_1, 3f_1, \dots, nf_1$$

The lowest characteristic frequency of vibration is the fundamental frequency, which corresponds to the first harmonic.

Quantization of Frequencies

Quantum jumps in frequency exist between the resonance frequencies. Stationary waves can be set up on the string only with the frequencies of the harmonic series, determined by the tension, the length and the mass per unit length. Waves which are not in the harmonic series are quickly damped out.

Tuning a Musical Instrument

The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. The tension in guitar and violin strings is varied by tightening the pegs on the neck, and once the instrument is tuned the musician varies the frequency by moving the fingers along the neck, thereby changing the length of the vibrating portion.

LQ 2 ends here

Harmonics

The set of all the possible standing waves of this series are called harmonics of the system.

$$f_1, 2f_1, 3f_1, \dots, nf_1$$

The lowest of all the harmonics is called the fundamental or first harmonic, and the subsequent frequencies are called the second harmonic, the third harmonic and so on.

Book Examples

Example 7.5.1 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency.

Given Data

$$\ell = 120 \text{ cm} = 1.2 \text{ m}$$

$$n = 4$$

$$f_4 = 120 \text{ Hz}$$

To Find

Wavelength $\lambda_4 = ?$ and fundamental frequency $f_1 = ?$

$$\text{Formula} \quad \lambda_n = \frac{2\ell}{n}$$

$$f_n = nf_1$$

Solution

The string vibrates in four segments and the distance between two consecutive nodes is half a wavelength, so

$$\lambda_4 = \frac{2 \times 1.2}{4} = 0.6 \text{ m}$$

The frequency of the fourth harmonic is four times the fundamental.

$$f_1 = \frac{120}{4} = 30 \text{ Hz}$$

Result

$$\lambda_4 = 0.6 \text{ m}$$

$$f_1 = 30 \text{ Hz}$$

Short Questions

SQ 7.5.1 On what does the speed of a transverse wave on a stretched string depend?

Two Factors

The tension F in the string and the mass per unit length m of the string.

$$v = \sqrt{\frac{F}{m}}$$

Independent Of

It does not depend on the point from which the string is plucked.

SQ 7.5.2 Why does a string clamped at both ends always carry a node at each end?

Ends Cannot Move

The clamps hold the two ends fixed, so no motion can take place there.

Consequence

A point that is permanently at rest is a node, so only those stationary waves can be set up that have a node at each end.

SQ 7.5.3 What is meant by the quantization of frequencies of a stretched string?

Discrete Frequencies

A string can be made to resonate only at the frequencies of the harmonic series.

$$f_n = nf_1$$

Meaning

There are quantum jumps between one resonance frequency and the next, fixed by the tension, the length and the mass per unit length, and any other frequency is quickly damped out.

SQ 7.5.4 In what two ways does a guitarist change the frequency of a string?

Changing the Tension

The pegs on the neck are tightened or loosened, which is how the instrument is tuned.

Changing the Length

Once tuned, the player moves the fingers along the neck and so changes the length of the vibrating portion.

SQ 7.5.5 A wire has a fundamental frequency of 256 Hz. What is the frequency of its third harmonic?

$$\text{Formula} \quad f_n = nf_1$$

$$\text{Calculation} \quad f_3 = 3 \times 256 = 768 \text{ Hz}$$

SQ 7.5.6 The tension in a stretched string is made four times as great. What happens to its fundamental frequency?

$$\text{Relation} \quad f_1 = \frac{1}{2\ell} \sqrt{\frac{F}{m}}$$
$$f_1 \propto \sqrt{F}$$

Effect

The square root of four is two, so the fundamental frequency is doubled.

SQ 7.5.7 Why are waves whose frequencies do not belong to the harmonic series quickly damped out on a string?

No Fit to the Ends

Such a wave cannot place a node at each clamped end.

Consequence

The wave and its reflection do not reinforce each other from one round trip to the next, so the vibration dies away instead of building into a stationary wave.

MCQ 7.5.1 The speed of a transverse wave on a stretched string is:

- | | |
|----------------|---------|
| A $\sqrt{Fm}$ | B F/m |
| C $\sqrt{F/m}$ | D m/F |

Solution: The speed rises with the tension and falls with the mass per unit length.

MCQ 7.5.2 A string clamped at both ends and plucked at its middle vibrates in:

- | | |
|---------------|-------------------|
| A two loops | B one loop |
| C three loops | D four loops |

Solution: Nodes form at the two clamped ends with a single antinode between them.

MCQ 7.5.3 A string plucked at one quarter of its length vibrates in:

- | | |
|--------------------|---------------|
| A one loop | B three loops |
| C two loops | D four loops |

Solution: The plucking point becomes an antinode, which places a node at the middle.

MCQ 7.5.4 If the fundamental frequency of a string is f_1 , its third harmonic is:

- | | |
|-----------|----------------------------|
| A $f_1/3$ | B $3f_1$ |
| C $2f_1$ | D $9f_1$ |

Solution: The harmonics are whole-number multiples of the fundamental.

MCQ 7.5.5 The wavelength of the n th mode on a string of length ℓ is:

- | | |
|-------------|-------------|
| A $2\ell/n$ | B $n\ell/2$ |
| C $\ell/2n$ | D $4\ell/n$ |

Solution: The string holds n loops, each of half a wavelength.

MCQ 7.5.6 The tension of a string is made four times as great. Its fundamental frequency becomes:

- | | |
|-----------------|-------------------------|
| A half as great | B four times as great |
| C unchanged | D twice as great |

Solution: The frequency varies as the square root of the tension.

MCQ 7.5.7 A string 1.5 m long carries waves of speed 300 m s^{-1} . Its fundamental frequency is:

- | | |
|-----------------|----------|
| A 100 Hz | B 50 Hz |
| C 200 Hz | D 450 Hz |

Solution: The fundamental is $v/2\ell = 300/3$.

MCQ 7.5.8 A wave whose frequency is not in the harmonic series of a string is:

- | | |
|--------------------------|-----------------------------|
| A amplified | B reflected without loss |
| C turned into a harmonic | D quickly damped out |

Solution: It cannot hold a node at each clamped end, so it dies away.

MCQ 7.5.9 A string vibrating in three loops carries how many nodes?

- | | |
|---------|---------------|
| A three | B four |
| C two | D six |

Solution: A string in n loops has $n + 1$ nodes, counting the two ends.

Topic 7.6 Stationary Waves in Air Columns

LQ 3 starts here — Find the frequencies of the harmonics produced in an organ pipe when it is open at both ends and when it is closed at one end.

Air Columns

Stationary waves can be set up in other media also, such as an air column inside a pipe or tube. A common example of a vibrating air column is the organ pipe.

Organ Pipe

An organ pipe is a wind instrument in which sound is produced due to the setting up of stationary waves in an

air column. It consists of a hollow long tube with both ends open, or with one end open and the other closed.

Reflecting End

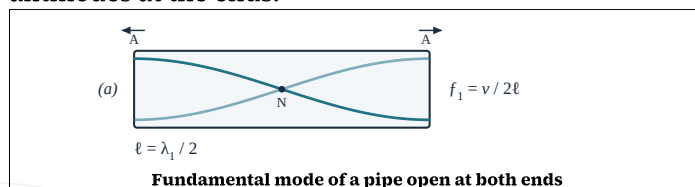
- If the reflecting end is open, the air molecules have complete freedom of motion and it behaves as an **antinode**.
- If the reflecting end is closed, it behaves as a **node**, because the movement of the molecules is restricted.

Pipe Open at Both Ends

Consider an organ pipe of length ℓ which is open at both ends, so that both ends act as antinodes. The longitudinal waves set up inside the pipe are represented by transverse curved lines, which show the displacement and amplitude of vibration of the air particles along the axis of the pipe.

Fundamental Mode

There is only one node at the middle of the pipe and two antinodes at the ends.



$$\ell = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2\ell$$

Fundamental Frequency

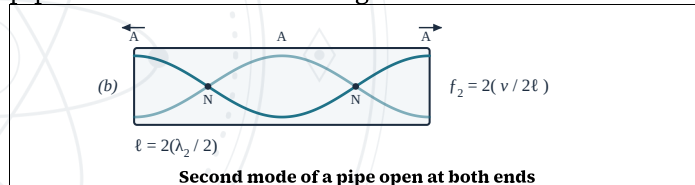
$$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{2\ell}$$

This frequency is called the fundamental frequency or the first harmonic.

Second Mode

With three antinodes and two nodes, the length of the pipe holds two half wavelengths.



$$\ell = \frac{2\lambda_2}{2}$$

$$\lambda_2 = \ell$$

Second Harmonic

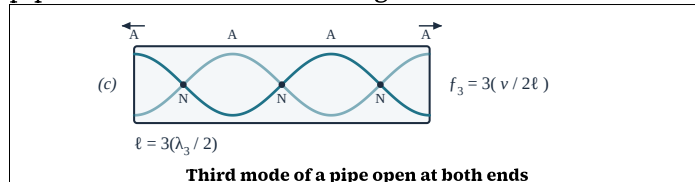
$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = \frac{v}{\ell} = 2 \left(\frac{v}{2\ell} \right) = 2f_1$$

The frequency is twice that of the fundamental.

Third Mode

With four antinodes and three nodes, the length of the pipe holds three half wavelengths.



n th Mode

For an air column vibrating in n loops

$$f_n = n \left(\frac{v}{2\ell} \right) = nf_1$$

and the wavelength is

$$\lambda_n = \frac{2\ell}{n}$$

Here n is any whole number, so the open pipe gives the whole harmonic series.

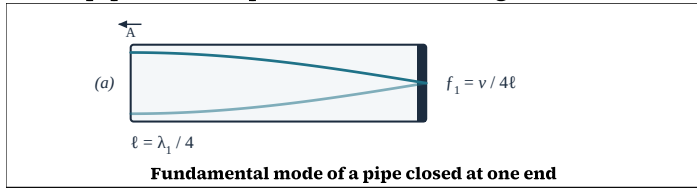
$$n = 1, 2, 3, 4, \dots$$

Pipe Closed at One End

Consider an organ pipe of length ℓ closed at one end. At the closed end there is a node and at the open end an antinode.

Fundamental Mode

The fundamental mode has one node and one antinode, so the pipe holds a quarter of a wavelength.



$$\ell = \frac{\lambda_1}{4}$$

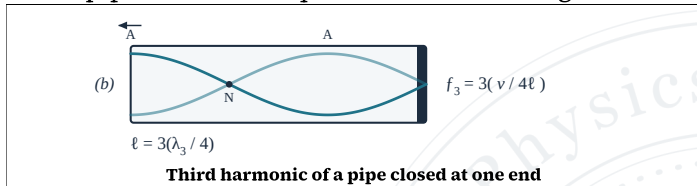
$$\lambda_1 = 4\ell$$

Fundamental Frequency $f_1 = \frac{v}{\lambda_1}$

$$f_1 = \frac{v}{4\ell}$$

Second Mode

The next mode contains two nodes and two antinodes, so the pipe holds three quarters of a wavelength.



$$\ell = \frac{3\lambda_3}{4}$$

$$\lambda_3 = \frac{4\ell}{3}$$

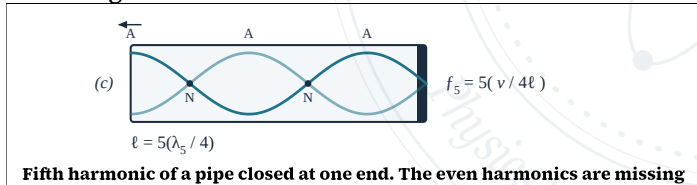
$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{3v}{4\ell} = 3f_1$$

Third Harmonic

Third Mode

The next mode after that contains three nodes and three antinodes, so the pipe holds five quarters of a wavelength.



nth Mode

For an air column vibrating in n loops

$$f_n = n \left(\frac{v}{4\ell} \right) = nf_1$$

and the wavelength is

$$\lambda_n = \frac{4\ell}{n}$$

Here n takes the odd values only, so only the odd harmonics are present.

$$n = 1, 3, 5, \dots$$

Conclusion

The pipe which is open at both ends is richer in harmonics than the pipe closed at one end.

LQ 3 ends here

Book Examples

Example 7.6.1 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is (a) open at both ends and (b) closed at one end. (Speed of sound = 350 m s^{-1})

Given Data

$$\ell = 50 \text{ cm} = 0.5 \text{ m}$$

$$v = 350 \text{ m s}^{-1}$$

To Find

Fundamental frequency and the next harmonic in each case

Formula

For a pipe open at both ends, with n any whole number

$$f_n = \frac{nv}{2\ell}$$

For a pipe closed at one end, with n odd only

$$f_n = \frac{nv}{4\ell}$$

Solution

Open at Both Ends

$$f_1 = \frac{1 \times 350}{2 \times 0.5} = 350 \text{ Hz}$$

$$f_2 = \frac{2 \times 350}{2 \times 0.5} = 700 \text{ Hz}$$

Closed at One End

$$f_1 = \frac{1 \times 350}{4 \times 0.5} = 175 \text{ Hz}$$

The next harmonic present is the third, because the even harmonics are missing.

$$f_3 = \frac{3 \times 350}{4 \times 0.5} = 525 \text{ Hz}$$

Result

$$f_1 = 350 \text{ Hz}$$

$$f_2 = 700 \text{ Hz}$$

$$f_1 = 175 \text{ Hz}$$

$$f_3 = 525 \text{ Hz}$$

Example 7.6.2 A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes. (Speed of sound = 340 m s^{-1})

Given Data

$$\ell_{\min} = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$$

$$\ell_{\max} = 4 \text{ m}$$

$$v = 340 \text{ m s}^{-1}$$

To Find

Frequency range of the fundamental notes

Formula

For a pipe open at one end only, the fundamental is obtained by putting $n = 1$

$$f_1 = \frac{v}{4\ell}$$

Solution

The shortest pipe gives the highest fundamental.

$$f_{\max} = \frac{340}{4 \times 30 \times 10^{-3}} = 2833.33 \text{ Hz}$$

The longest pipe gives the lowest fundamental.

$$f_{\min} = \frac{340}{4 \times 4} = 21.25 \text{ Hz}$$

Result

The fundamental frequency range is approximately from 21 Hz to 2833 Hz.

Short Questions

SQ 7.6.1 Why is the open end of an organ pipe an antinode and the closed end a node?

Open End

The air molecules there have complete freedom of motion, so the displacement is a maximum and the end behaves as an antinode.

Closed End

The movement of the molecules is restricted by the closed end, so the displacement is zero and the end behaves as a node.

SQ 7.6.2 Why is a pipe open at both ends richer in harmonics than a pipe closed at one end?

Open Pipe

Every whole number is allowed.

$$f_n = n \left(\frac{v}{2\ell} \right)$$

Closed Pipe

Only the odd numbers are allowed.

$$f_n = n \left(\frac{v}{4\ell} \right)$$

Conclusion

The closed pipe is missing all its even harmonics, so the open pipe sounds the richer of the two.

SQ 7.6.3 Compare the fundamental frequencies of an open pipe and a closed pipe of the same length.

Open Pipe $f_1 = \frac{v}{2\ell}$

Closed Pipe $f_1 = \frac{v}{4\ell}$

Comparison

The closed pipe sounds a fundamental of half the frequency of the open pipe of the same length, that is, an octave lower.

SQ 7.6.4 The fundamental frequency of a pipe closed at one end is 100 Hz. Write the frequencies of the next two notes it can produce.

Odd Harmonics Only $f_n = n f_1$

Calculation

$$f_3 = 3 \times 100 = 300 \text{ Hz}$$

$$f_5 = 5 \times 100 = 500 \text{ Hz}$$

SQ 7.6.5 Why are the longitudinal stationary waves inside a pipe drawn as transverse curved lines in the book's figures?

A Drawing Convention

The air particles actually move to and fro along the axis of the pipe, which cannot be drawn along that same axis.

What the Curve Shows

The curved lines stand for the displacement and the amplitude of the air particles at each point along the pipe, not for a sideways motion.

SQ 7.6.6 A pipe closed at one end is 0.25 m long. Find its fundamental frequency, taking the speed of sound as 340 m s^{-1} .

Formula $f_1 = \frac{v}{4\ell}$

Calculation $f_1 = \frac{340}{4 \times 0.25} = 340 \text{ Hz}$

SQ 7.6.7 Which harmonics are missing from a pipe closed at one end, and why?

Missing

All the even harmonics are missing.

Reason

The pipe must hold a node at the closed end and an antinode at the open end, and only an odd number of quarter wavelengths can do that.

$$\ell = \frac{n\lambda_n}{4} \quad n = 1, 3, 5, \dots$$

MCQ 7.6.1 At the open end of an organ pipe there is always:

- | | |
|----------------------|---------------------|
| A a node | B zero displacement |
| C an antinode | D no vibration |

Solution: The air molecules there move freely, giving maximum displacement.

MCQ 7.6.2 At the closed end of an organ pipe there is always:

- | | |
|-----------------|------------------------|
| A an antinode | B maximum displacement |
| C a node | D no reflection |

Solution: The closed end restricts the motion of the molecules, so the displacement is zero.

MCQ 7.6.3 For a pipe of length ℓ open at both ends, the fundamental wavelength is:

- | | |
|-----------|------------|
| A 2ℓ | B ℓ |
| C 4ℓ | D $\ell/2$ |

Solution: The pipe holds one half wavelength between its two end antinodes.

MCQ 7.6.4 For a pipe of length ℓ closed at one end, the fundamental wavelength is:

- | | |
|-----------|------------|
| A 2ℓ | B 4ℓ |
| C ℓ | D $\ell/4$ |

Solution: The pipe holds a quarter of a wavelength between the node and the antinode.

MCQ 7.6.5 A pipe closed at one end produces:

- | | |
|---------------------------|---------------------------------|
| A all the harmonics | B only the odd harmonics |
| C only the even harmonics | D no harmonics |

Solution: Only an odd number of quarter wavelengths fits between a node and an antinode.

MCQ 7.6.6 The fundamental frequency of a closed pipe compared with an open pipe of the same length is:

- | | |
|------------------------|-----------------------|
| A twice as great | B four times as great |
| C half as great | D equal |

Solution: The closed pipe needs four times the length per wavelength, so its fundamental is $v/4\ell$ against $v/2\ell$.

MCQ 7.6.7 An organ pipe open at both ends is 0.5 m long. With the speed of sound 340 m s^{-1} , its fundamental frequency is:

- | | |
|----------|-----------------|
| A 170 Hz | B 680 Hz |
| C 85 Hz | D 340 Hz |

Solution: The fundamental is $v/2\ell = 340/1.0$.

MCQ 7.6.8 A pipe closed at one end has a fundamental of 150 Hz. The next note it can sound is:

- | | |
|-----------------|----------|
| A 450 Hz | B 300 Hz |
| C 600 Hz | D 900 Hz |

Solution: The even harmonics are missing, so the next is the third, 3×150 .

MCQ 7.6.9 The pipe that is richer in harmonics is the one:

- | | |
|----------------------------|---------------------|
| A open at both ends | B closed at one end |
| C of larger diameter | D of shorter length |

Solution: The open pipe sounds every whole-number multiple of its fundamental.

Topic 7.7 Experiment Demonstrating Stationary Waves Using Microwaves

Microwaves

Microwaves are a form of electromagnetic radiation. They are called micro waves because their wavelengths are typically of the order of millimetres or centimetres, much shorter than radio waves.

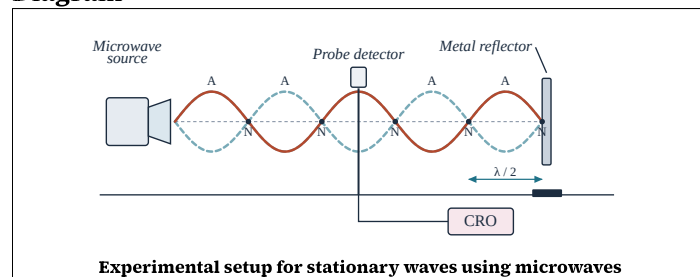
Stationary Microwaves

Stationary waves can be produced by microwaves when they are confined to a specific region or cavity, such as a wave guide or a resonant chamber. In these structures the microwaves bounce back and forth, creating a standing wave pattern with nodes and antinodes. It occurs when the microwave frequency matches the resonant frequency of the cavity.

Apparatus

The experimental setup consists of a microwave source, which is the transmitter, a probe detector, and a metal reflector, which is a metallic plate that reflects the microwaves. The three are placed in a line.

Diagram



Procedure

- The waves coming out of the microwave source move towards the metal plate and are reflected back.
- The reflected wave and the incident wave superpose and create a stationary wave pattern.

- The pattern is detected by the probe detector placed between the transmitter and the metallic plate, which shows the intensity of the signal.
- Moving the plate or the detector locates the antinodes and the nodes.

Result

The distance from one antinode to the next is half a wavelength, so measuring it gives the wavelength of the stationary wave.

Short Questions

SQ 7.7.1 Why are microwaves given that name?

Short Wavelength

Their wavelengths are typically of the order of millimetres or centimetres.

Comparison

That is much shorter than the wavelength of radio waves, and the prefix records how small they are.

SQ 7.7.2 In the microwave experiment, how is the wavelength of the microwaves found?

Locating the Antinodes

The probe detector is moved between the transmitter and the metal plate, and the positions of maximum signal are noted.

Measurement

The distance from one antinode to the next is half a wavelength, so measuring it gives the wavelength.

$$\lambda = 2 \times (\text{antinode to antinode})$$

SQ 7.7.3 What is the function of the metal reflector in the microwave experiment?

Returns the Wave

It reflects the microwaves straight back along the path they came.

Why It Is Needed

The reflected wave superposes with the incident wave, and two waves travelling in opposite directions are what a stationary wave needs.

MCQ 7.7.1 The wavelengths of microwaves are typically of the order of:

- | | |
|------------------------------|--------------|
| A kilometres | B metres |
| C millimetres or centimetres | D nanometres |

Solution: That short wavelength is what the prefix "micro" records.

MCQ 7.7.2 In the microwave experiment the stationary wave is formed by the incident wave together with the wave reflected from the:

- | | |
|---------------|-------------------|
| A metal plate | B probe detector |
| C transmitter | D surrounding air |

Solution: The metallic plate sends the wave back along its own path.

MCQ 7.7.3 Stationary microwaves are set up in a cavity when the microwave frequency matches the cavity's:

- | | |
|----------------------|------------------|
| A temperature | B wall thickness |
| C resonant frequency | D power rating |

Solution: At the resonant frequency the reflected waves reinforce one another inside the cavity.

MCQ 7.7.4 The distance from one antinode to the next, found with the probe, is equal to:

- | | |
|-------------------------------|-----------------------|
| A the wavelength | B half the wavelength |
| C a quarter of the wavelength | D the frequency |

Solution: Antinodes recur every half wavelength, which is how the wavelength is measured.

Topic 7.8 Diffraction of Waves

LQ 4 starts here — Define and exemplify diffraction of waves. Describe this phenomenon by ripple tank experiment.

Diffraction

Diffraction of waves is the bending of waves around the sharp edges or corners of obstacles, or the spreading of waves beyond a barrier.

When It Occurs

It occurs when a wave encounters a physical barrier or an opening that is comparable in size to the wavelength of the wave. The longer the wavelength, the greater the spreading, and the shorter the wavelength, the less the spreading.

Where It Is Seen

Diffraction can be observed in water waves, sound waves, light waves and electromagnetic waves.

Examples

- Sound waves are heard around corners and through a doorway, because they bend around the corners.
- X-rays are diffracted by crystals, because the spacing between the regular arrays of atoms is of the order of the X-ray wavelength.

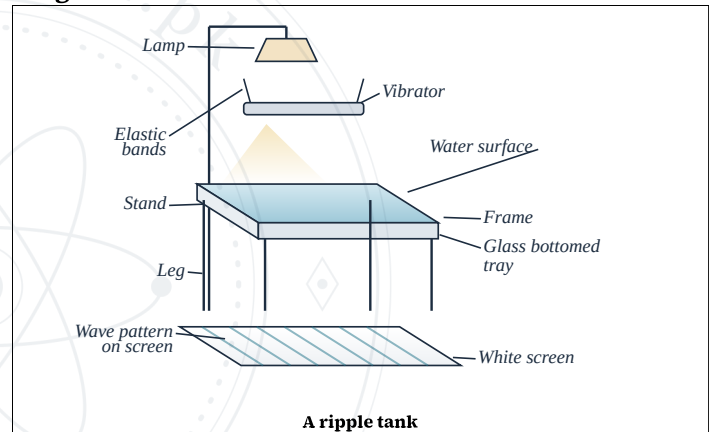
The Ripple Tank

A ripple tank is a very useful apparatus, not only to generate water waves but also to demonstrate wave properties such as reflection, refraction and diffraction.

Parts

It contains water, a vibrator such as a motorized oscillating needle, obstacles such as a small rectangular block or a semicircular barrier, and gap widths of different sizes.

Diagram



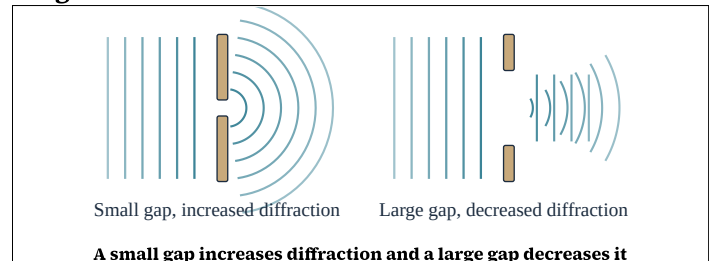
Procedure

The vibrator creates a series of concentric circles or parallel waves. An obstacle is placed so as to create a gap of a specific width, and the experiment is repeated with different gap widths.

Observation

- When the gap width is small compared to the wavelength, diffraction is significant and the waves bend around the obstacle, creating a semicircular pattern.
- As the gap width increases relative to the wavelength, diffraction decreases and the waves pass through the gap with less bending.

Diagram



Conclusion

The experiment demonstrates the qualitative effect of gap width on diffraction, illustrating how the relationship between the gap width and the wavelength affects the behaviour of the wave.

LQ 4 ends here

Book Examples

Example 7.8.1 In a ripple tank, a wave generator produces 500 pulses in 10 s. Find the frequency and the time period of the pulses produced.

Given Data $N = 500$ pulses
 $t = 10$ s

To Find
Frequency $f = ?$ and time period $T = ?$

Formula $f = \frac{N}{t}$
 $T = \frac{1}{f}$

Solution
 $f = \frac{500}{10} = 50$ Hz
 $T = \frac{1}{50} = 0.02$ s

Result $f = 50$ Hz
 $T = 0.02$ s

Short Questions

SQ 7.8.1 How does the width of the gap, compared with the wavelength, decide how much diffraction is seen?

Narrow Gap

When the gap is small compared with the wavelength, diffraction is significant and the waves spread beyond it in a semicircular pattern.

Wide Gap

As the gap grows wide compared with the wavelength, diffraction decreases and the waves pass through with little bending.

SQ 7.8.2 Why are X-rays diffracted by crystals?

Matching Sizes

Diffraction needs an obstacle or opening comparable in size with the wavelength.

In a Crystal

The spacing between the regular arrays of atoms is of the order of the X-ray wavelength, so the crystal acts as the grating.

SQ 7.8.3 Why can a person be heard from round a corner although they cannot be seen?

Sound Diffracts Strongly

The wavelength of sound is comparable with the width of a doorway, so sound waves bend round the corner and reach the ear.

Light Does Not

The wavelength of light is far smaller than the doorway, so the spreading is too small to notice and the light travels on in straight lines.

SQ 7.8.4 In a ripple tank the barrier is changed for one with a narrower gap. How does the pattern beyond the gap change?

More Spreading

The narrower gap makes the diffraction greater.

Observation

The straight waves that fall on the barrier leave the gap as a semicircular pattern spreading into the whole region behind it.

MCQ 7.8.1 Diffraction at a gap is greatest when the gap width is:

- | | | | |
|---|----------------------------------|---|--|
| A | much greater than the wavelength | B | comparable with or smaller than the wavelength |
| C | exactly twice the wavelength | D | independent of the wavelength |

Solution: A gap of about the size of the wavelength spreads the waves into a semicircular pattern.

MCQ 7.8.2 Of two waves passing through the same gap, the one that spreads more is the one with the:

- | | | | |
|---|------------------|---|-------------------|
| A | greater speed | B | higher frequency |
| C | larger amplitude | D | longer wavelength |

Solution: The longer the wavelength, the greater the spreading.

MCQ 7.8.3 X-rays are diffracted by a crystal because the spacing of its atoms is of the order of the:

- | | | | |
|---|---------------------|---|-----------------------|
| A | X-ray frequency | B | X-ray wavelength |
| C | size of the crystal | D | radius of an electron |

Solution: Diffraction needs an obstacle comparable in size with the wavelength.

MCQ 7.8.4 The vibrator of a ripple tank produces:

- | | | | |
|---|---------------------|---|--------------------------------------|
| A | only straight waves | B | stationary waves only |
| C | sound waves | D | concentric circles or parallel waves |

Solution: A motorized oscillating needle or bar generates circular or plane water waves.

MCQ 7.8.5 As the gap width increases relative to the wavelength, the diffraction:

- | | | | |
|---|------------------|---|----------------|
| A | increases | B | stays the same |
| C | becomes infinite | D | decreases |

Solution: The waves then pass through with less bending.

MCQ 7.8.6 Hearing a sound from round a corner is an everyday example of:

- | | | | |
|---|------------|---|--------------|
| A | refraction | B | interference |
| C | resonance | D | diffraction |

Solution: The sound bends around the sharp edge of the corner.

Topic 7.9 Beats

LQ 5 starts here — What is meant by the term beats? Prove that number of beats per second is equal to the difference between the frequencies of vibrating tuning forks.

Beats

When two waves of slightly different frequencies, travelling in the same direction, overlap each other, there is a periodic variation of sound between maximum and minimum loudness, which is called beats.

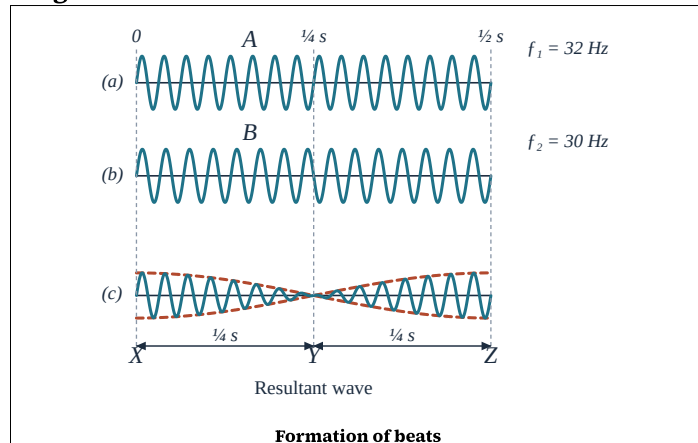
Two Identical Tuning Forks

Tuning forks give out pure notes of a single frequency. If two tuning forks *A* and *B* of the same frequency, say 32 Hz, are sounded separately, each gives out a pure note. Sounded together, their sound waves superpose and are heard by the ear as a single pure note.

Loading One Fork

If the tuning fork *B* is loaded with some wax or plasticine, its frequency is lowered slightly, say to 30 Hz. Sounded together now, the two forks give a note of alternately increasing and decreasing intensity. This note is called a beat, and it is due to interference between the sound waves from the two forks.

Diagram



Explanation of the Waveform

- At the instant X the displacements of the two waves are in the same direction, the resultant displacement is large and a loud sound is heard.
- After $1/4$ s the displacement due to one fork is opposite to that due to the other, the resultant displacement is minimum at Y , and a faint sound or no sound is heard.
- Another $1/4$ s later the displacements are again in the same direction and a loud sound is heard at Z .

Counting the Beats

During one second two loudest sounds and two faintest sounds are observed, and the difference of the frequencies of the two tuning forks is also 2 Hz.

$$f_A = 32 \text{ Hz}$$

$$f_B = 30 \text{ Hz}$$

$$\text{beats} = 32 - 30 = 2$$

Beat Frequency

The number of beats per second is equal to the difference between the frequencies of the two tuning forks.

$$f_b = f_A - f_B$$

Limit of Detection

When the difference between the frequencies of the two sounds is more than 10 Hz, it becomes difficult to recognize the beats.

LQ 5 ends here

Tuning by Beats

Beats are used to tune a string instrument such as a piano or a violin, by beating a note against a note of known frequency. The string is then tightened or loosened until no beats are heard.

Beats in Musical Instruments

- Guitar: playing a standard tuned string together with a string tuned a few cents higher or lower creates beats.
- Piano: playing two keys adjacent to each other creates beats.
- Violin: playing two strings with slightly different bow pressures or speeds generates beats.
- Drums: two drums of slightly different tunings played together create beats.
- Flute: two notes played with slightly different embouchure positions generate beats.
- Organ: two pipes of slightly different tunings create beats.
- Synthesizer: two oscillators of slightly different frequencies create beats.

Use in Music

Musicians often use beats intentionally to create rhythmic effects, add texture, or produce a sense of tension and release. Where the beats are unwanted, the tuning, pitch or playing technique is adjusted to minimize them.

Book Examples

Example 7.9.1 Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prongs, and the two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

Given Data

$$f_1 = 256 \text{ Hz}$$

$$f_b = 3 \text{ Hz}$$

$$f'_b = 1 \text{ Hz after loading}$$

To Find

Frequency of the second fork $f_2 = ?$

$$\text{Formula} \quad f_b = f_1 - f_2$$

$$f_2 = f_1 \pm f_b$$

Solution

Putting the values, the second fork is either

$$f_2 = 256 + 3 = 259 \text{ Hz}$$

or

$$f_2 = 256 - 3 = 253 \text{ Hz}$$

Choosing Between Them

Loading the first fork with wax lowers its frequency below 256 Hz. If the second fork were 259 Hz the gap between them would widen and the beats would rise above 3 per second. The beats fell to 1 per second instead, so the second fork must lie below the first.

$$\text{Result} \quad f_2 = 253 \text{ Hz}$$

Short Questions

SQ 7.9.1 Two tuning forks of 256 Hz and 260 Hz are sounded together. How many beats are heard in one second?

$$\text{Formula} \quad f_b = f_1 - f_2$$

$$\text{Calculation} \quad f_b = 260 - 256 = 4$$

Answer

Four beats are heard each second.

SQ 7.9.2 What happens to the frequency of a tuning fork when it is loaded with wax, and how does that settle which of two possible answers is right?

Frequency Falls

The wax lowers the frequency of the fork slightly.

Use

A fork beating with a standard could be above or below it. Loading one fork and listening again decides the case: if the beats grow faster the loaded fork was the higher of the two, and if they slow down it was the lower.

SQ 7.9.3 How are beats used to tune a musical instrument?

Beating Against a Standard

The note of the string is sounded together with a note of known frequency.

Adjustment

The string is tightened or loosened until no beats are heard, and the two frequencies are then equal.

SQ 7.9.4 In one second a listener hears two loudest sounds and two faintest sounds. What is the difference between the frequencies of the two sources?

Counting

Two loud sounds in one second means two beats per second.

Answer

The number of beats per second is the difference of the two frequencies, so the difference is 2 Hz.

SQ 7.9.5 Give three examples of musical instruments in which beats are generated, and say how.

Guitar

A standard tuned string is played together with a string tuned a few cents higher or lower.

Piano

Two keys adjacent to each other are played together.

Organ

Two pipes of slightly different tunings are sounded together.

MCQ 7.9.1 Beats are produced by two waves of:

A exactly the same frequency

B **slightly different frequencies**

C opposite phase

D different amplitudes

Solution: The small difference in frequency makes the resultant swell and fade.

MCQ 7.9.2 The number of beats heard per second is equal to:

- | | |
|---|--|
| A the sum of the two frequencies | B the difference of the two frequencies |
| C the product of the two frequencies | D the mean of the two frequencies |

Solution: Each beat marks one cycle by which one wave has gained on the other.

MCQ 7.9.3 Two forks of 300 Hz and 304 Hz sounded together give:

- | | |
|---------------------------------|-------------------------------|
| A four beats per second | B two beats per second |
| C eight beats per second | D 604 beats per second |

Solution: The beat frequency is the difference, 304 – 300.

MCQ 7.9.4 Loading a tuning fork with wax:

- | | |
|-------------------------------|------------------------------|
| A raises its frequency | B leaves it unchanged |
| C lowers its frequency | D stops it vibrating |

Solution: The added wax lowers the frequency slightly, which is how the book separates the two possible answers.

MCQ 7.9.5 A piano string is in tune with the standard note when:

- | | |
|--------------------------------|--|
| A the beats are loudest | B the beats number ten per second |
| C no beats are heard | D the note disappears |

Solution: Equal frequencies give no beats at all.

MCQ 7.9.6 A fork of 256 Hz gives three beats per second with a second fork. The second fork is:

- | | |
|---------------------------|----------------------|
| A 250 Hz only | B 262 Hz only |
| C 259 Hz or 253 Hz | D 256 Hz |

Solution: The difference of three may be either way, so a further test is needed to decide.

MCQ 7.9.7 Two adjacent keys of a piano played together produce beats because their frequencies are:

- | | |
|-----------------------------|-------------------------------|
| A equal | B in the ratio 2 to 1 |
| C slightly different | D completely unrelated |

Solution: Neighbouring notes differ by a small amount, which is heard as a pulsation.

Topic 7.10 Intensity of a Wave

LQ 6 starts here – What do you understand by progressive waves? Discuss the intensity of progressive waves.

Intensity

Intensity is defined as the amount of energy transmitted per unit area per unit time in the direction of propagation of a progressive wave.

Meaning

It is a measure of the power of a wave. It is denoted by I and is measured in watts per square metre.

Progressive Wave

A progressive or travelling wave is one that travels through a medium in a consistent direction, transferring energy from one point to another. It moves forward, as opposed to a stationary wave. Water waves, sound waves and light waves are progressive waves.

Formula

By definition, the intensity is the energy per unit area per unit time.

$$I = \frac{E}{At}$$

Since the energy transmitted per unit time is the power

$$I = \frac{P}{A}$$

Here I is the intensity in W m^{-2} and E is the energy in joule. The time t is in second, the area A in m^2 and the power P in watt.

Spherical Spreading

Where a source radiates evenly in all directions, the area is the surface area of a sphere of radius r

$$A = 4\pi r^2$$

$$I = \frac{P}{4\pi r^2}$$

Intensity and Amplitude

In mechanical waves such as sound waves, water waves or waves on a vibrating string, energy is stored as the kinetic energy and the potential energy of the particles of the medium. How much energy is stored depends upon the displacement of the particles from the mean position.

Relation

The intensity of a wave is therefore proportional to the square of the amplitude.

$$\frac{I}{A^2} = \text{constant}$$

$$I \propto A^2$$

Hence

$$I = kA^2$$

Here k is the constant of proportionality and depends upon the physical properties of the wave and the medium.

Consequence

Doubling the amplitude of a wave makes its intensity four times, and tripling the amplitude makes the intensity nine times.

Adding Intensities

When two progressive waves fall on the same area, the total intensity is the sum of the individual intensities.

$$I = I_1 + I_2$$

LQ 6 ends here

Book Examples

Example 7.10.1 (a) A wave has an intensity of 0.5 W m^{-2} at a distance of 3.0 m from the source. What is the power of the wave? (b) Two progressive waves have intensities of 0.5 W m^{-2} and 0.25 W m^{-2} . Find the total intensity of the two waves.

Given Data

$$I = 0.5 \text{ W m}^{-2}$$

$$r = 3.0 \text{ m}$$

$$I_1 = 0.5 \text{ W m}^{-2}$$

$$I_2 = 0.25 \text{ W m}^{-2}$$

To Find

Power $P = ?$ and total intensity $I = ?$

Formula $I = \frac{P}{4\pi r^2}$

$$P = 4\pi r^2 I$$

Solution

The wave spreads over the surface of a sphere of radius 3.0 m.

$$P = 4 \times 3.14 \times (3.0)^2 \times 0.5 = 56.52 \text{ W}$$

The intensities of two waves falling on the same area add up.

$$I = 0.5 + 0.25 = 0.75 \text{ W m}^{-2}$$

Result

$$P = 56.52 \text{ W}$$

$$I = 0.75 \text{ W m}^{-2}$$

Example 7.10.2 A speaker is emitting sound waves with a power of 50 watts. The sound waves spread out evenly in all directions, and the surface area of the sphere at a distance of 5 m from the speaker is approximately 314 m^2 . What is the intensity of the sound waves at that distance?

Given Data $P = 50 \text{ W}$

$$A = 314 \text{ m}^2$$

To Find Intensity $I = ?$

Formula $I = \frac{P}{A}$

Solution $I = \frac{50}{314} = 0.159 \text{ W m}^{-2}$

Result $I = 0.159 \text{ W m}^{-2}$

Short Questions

SQ 7.10.1 Define the intensity of a wave and give its SI unit.

Intensity

The amount of energy transmitted per unit area per unit time in the direction of propagation of a progressive wave.

$$I = \frac{E}{At}$$

Unit

It is measured in watt per square metre.

SQ 7.10.2 Why does the intensity of a wave from a point source fall off as the distance from the source increases?

Same Power, Larger Area

The source sends out a fixed power, which spreads over the surface of a sphere.

$$I = \frac{P}{4\pi r^2}$$

Consequence

The area grows as the square of the distance, so the intensity falls as the inverse square of the distance.

SQ 7.10.3 On what does the constant k in the relation between intensity and amplitude depend?

Relation $I = kA^2$

The Constant

It depends upon the physical properties of the wave and of the medium through which it travels, and so has a different value for a different wave or a different medium.

SQ 7.10.4 Two progressive waves of intensities 0.8 W m^{-2} and 0.4 W m^{-2} fall on the same area. Find the total intensity.

Formula $I = I_1 + I_2$

Calculation $I = 0.8 + 0.4 = 1.2 \text{ W m}^{-2}$

SQ 7.10.5 A lamp radiates 100 W evenly in all directions. Find the intensity of the light 2.0 m away.

Formula $I = \frac{P}{4\pi r^2}$

Calculation $I = \frac{100}{4 \times 3.14 \times (2.0)^2} = 1.99 \text{ W m}^{-2}$

MCQ 7.10.1 The SI unit of the intensity of a wave is:

- | | |
|---------|--------------------------------|
| A watt | B watt per square metre |
| C joule | D joule per second |

Solution: Intensity is power per unit area.

MCQ 7.10.2 Intensity is the energy transmitted per unit area per unit:

- | | |
|----------|---------------|
| A length | B volume |
| C mass | D time |

Solution: Energy per unit area per unit time is power per unit area.

MCQ 7.10.3 For a source radiating evenly in all directions, the intensity at a distance r is:

- | | |
|----------------|----------------|
| A P/r^2 | B $P/2\pi r^2$ |
| C $P/4\pi r^2$ | D $P/4\pi r$ |

Solution: The power is spread over the surface of a sphere of radius r .

MCQ 7.10.4 If the amplitude of a wave is doubled, its intensity becomes:

- | | |
|---------------------|----------|
| A four times | B double |
| C eight times | D half |

Solution: Intensity varies as the square of the amplitude.

MCQ 7.10.5 Two waves of intensities 0.6 W m^{-2} and 0.3 W m^{-2} fall on the same area. The total intensity is:

- | | |
|--|---------------------------|
| A 0.9 W m^{-2} | B 0.3 W m^{-2} |
| C 1.8 W m^{-2} | D 0.18 W m^{-2} |

Solution: The intensities of the two waves add.

MCQ 7.10.6 A 200 W source spread over an area of 50 m^2 gives an intensity of:

- | | |
|-------------------------|--|
| A 2 W m^{-2} | B 4 W m^{-2} |
| C 10 W m^{-2} | D 250 W m^{-2} |

Solution: The intensity is the power divided by the area.

MCQ 7.10.7 A wave that travels through a medium in a consistent direction, carrying energy forward, is called:

- | | |
|---------------------|-----------------------------|
| A a stationary wave | B a progressive wave |
| C a standing wave | D a node |

Solution: A progressive or travelling wave moves forward, unlike a standing wave.

Topic 7.11 Doppler Effect

LQ 7 starts here — Keeping in mind the Doppler effect, analyze the following cases: (a) when source of sound moves away from the stationary observer (b) when source of sound moves towards the stationary observer.

Doppler Effect

The apparent change in the frequency, or pitch, of waves due to the relative motion between the source and the observer is called the Doppler effect.

Discovery

The effect was first observed by John Doppler while he was observing the frequency of light emitted from a star. The frequency of the emitted light was slightly different from that emitted by a similar source on the Earth, and he found that the change depends upon the motion of the star relative to the Earth.

With Sound

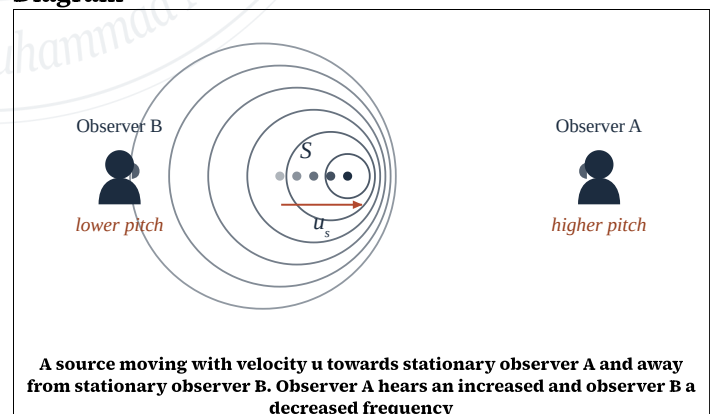
When an observer stands on a railway platform, the pitch of the whistle of an engine coming towards the platform appears to become higher, while the pitch of the whistle of an engine going away from the platform appears to become lower.

Stationary Source

Consider a source of sound S at rest emitting waves of wavelength λ , and let the speed of sound for a stationary observer be v . The number of waves received by the observer in one second is

$$f = \frac{v}{\lambda}$$

Diagram



Source Moving Towards a Stationary Observer

When the source moves towards the stationary observer A with velocity u_s , the waves are compressed and their wavelength is decreased, because the same number of waves is contained in a shorter space.

Doppler Shift

$$\Delta\lambda = \frac{u_s}{f}$$

Here $\Delta\lambda$ is the decrease in wavelength in one second and is called the Doppler shift.

Observed Wavelength

$$\lambda_A = \lambda - \Delta\lambda$$
$$\lambda_A = \frac{v}{f} - \frac{u_s}{f} = \frac{v - u_s}{f}$$

Apparent Frequency

$$f_A = \frac{v}{\lambda_A}$$
$$f_A = \left(\frac{v}{v - u_s} \right) f$$

Conclusion

The factor in the bracket is greater than one, so the apparent frequency is greater than the real frequency. The apparent frequency of the sound heard by the observer increases, which in turn increases the pitch of the sound.

Source Moving Away From a Stationary Observer

When the source moves away from the stationary observer B with velocity u_s , the waves are expanded and their wavelength is increased, because the same number of waves is now contained in a larger distance.

Doppler Shift

$$\Delta\lambda = \frac{u_s}{f}$$

Here $\Delta\lambda$ is the increase in wavelength in one second.

Observed Wavelength

$$\lambda_B = \lambda + \Delta\lambda$$
$$\lambda_B = \frac{v}{f} + \frac{u_s}{f} = \frac{v + u_s}{f}$$

Apparent Frequency

$$f_B = \frac{v}{\lambda_B}$$
$$f_B = \left(\frac{v}{v + u_s} \right) f$$

Conclusion

The factor in the bracket is less than one, so the apparent frequency is less than the real frequency. The apparent frequency of the sound heard by the observer decreases, which in turn decreases the pitch of the sound.

LQ 7 ends here

Book Examples

Example 7.11.1 Two trucks P and Q travel along a motorway in the same direction. The leading truck P travels at a steady speed of 12 m s^{-1} and the other truck Q , travelling at a steady speed of 20 m s^{-1} , sounds its horn to emit a steady note which P 's driver estimates has a frequency of 830 Hz . What frequency does Q 's own driver hear? (Speed of sound = 340 m s^{-1})

Given Data

$$u_P = 12 \text{ m s}^{-1}$$
$$u_Q = 20 \text{ m s}^{-1}$$
$$v = 340 \text{ m s}^{-1}$$
$$f_A = 830 \text{ Hz}$$

To Find

Frequency heard by Q 's own driver $f = ?$

Formula

The source approaches the listener, so

$$f_A = \left(\frac{v}{v - u_s} \right) f$$
$$f = \left(\frac{v - u_s}{v} \right) f_A$$

Solution

Only the relative motion of the two trucks matters, so the speed of the source is the speed of Q relative to P

$$u_s = 20 - 12 = 8 \text{ m s}^{-1}$$

Putting the values in the formula

$$f = \left(\frac{340 - 8}{340} \right) \times 830 = 810.47 \text{ Hz}$$

Result $f = 810.47 \text{ Hz}$

Meaning

The driver of Q hears the true note of his own horn, which is lower than the note P 's driver hears.

Example 7.11.2 A train sounds its horn before it sets off from a station and an observer waiting on the platform estimates its frequency at 1200 Hz . The train then moves off and accelerates steadily. Fifty seconds after departure the driver sounds the horn again and the platform observer estimates the frequency at 1140 Hz . Calculate the speed of the train 50 s after departure, and how far from the station the train is after 50 s . (Speed of sound = 340 m s^{-1})

Given Data

$$f = 1200 \text{ Hz}$$
$$f' = 1140 \text{ Hz}$$
$$v = 340 \text{ m s}^{-1}$$
$$t = 50 \text{ s}$$

To Find

Speed of the train $u_s = ?$ and distance covered $S = ?$

Formula

The source recedes from the observer, so

$$f' = \left(\frac{v}{v + u_s} \right) f$$

The train starts from rest and accelerates steadily, so

$$S = \left(\frac{v_i + v_f}{2} \right) t$$

Solution

Putting the values in the Doppler formula

$$340 + u_s = \frac{340 \times 1200}{1140} = 357.89 \text{ m s}^{-1}$$

$$u_s = 357.89 - 340 = 17.89 \text{ m s}^{-1}$$

The distance covered in 50 s is

$$S = \left(\frac{0 + 17.89}{2} \right) \times 50 = 448 \text{ m}$$

Result

$$u_s = 17.89 \text{ m s}^{-1}$$
$$S = 448 \text{ m}$$

Short Questions

SQ 7.11.1 What is meant by the Doppler shift?

Doppler Shift

It is the change in the wavelength of the waves in one second, caused by the motion of the source.

$$\Delta\lambda = \frac{u_s}{f}$$

Sign

It is a decrease in wavelength when the source approaches and an increase when the source recedes.

SQ 7.11.2 Why is the wavelength shorter in front of a source that is moving forward?

Same Waves, Shorter Space

The source emits the same number of waves each second, but it has moved forward while doing so.

Consequence

Those waves are packed into a shorter distance ahead of the source, so each one is shorter and the observed frequency is higher.

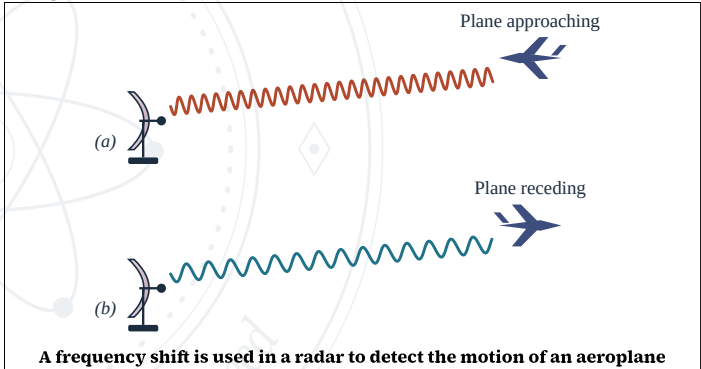
SQ 7.11.3 A source of frequency 500 Hz moves towards a stationary listener at 34 m s^{-1} . Find the frequency heard, taking the speed of sound as 340 m s^{-1} .

$$\text{Formula} \quad f_A = \left(\frac{v}{v - u_s} \right) f$$

Calculation

$$f_A = \left(\frac{340}{340 - 34} \right) \times 500 = 555.6 \text{ Hz}$$

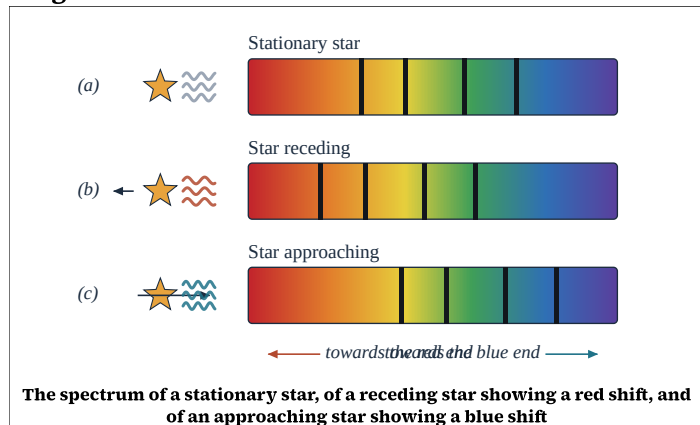
SQ 7.11.4 The whistle of an engine seems to drop in pitch as the engine passes the platform. Explain.	
Approaching While the engine comes towards the platform the waves are compressed, the observed frequency is higher than the true one and the pitch sounds <u>high</u> . Receding Once it has passed, the waves are stretched out, the observed frequency is lower than the true one and the pitch sounds low. The listener hears the change at the moment of passing.	
SQ 7.11.5 What is assumed about the observer in deriving the two Doppler formulas of this section?	
Observer at Rest The observer is taken to be <u>stationary</u> , and only the source moves. Why It Matters The formulas alter the wavelength that arrives at the observer, and that is the whole of the effect only while the observer is not moving through the waves as well.	
SQ 7.11.6 Write the factor by which the true frequency is multiplied when the source recedes, and say whether it is greater or less than one.	
Factor $f_B = \left(\frac{v}{v + u_s} \right) f$ Size The denominator is the larger, so the factor is <u>less than one</u> and the apparent frequency is lower than the true frequency.	
MCQ 7.11.1 The Doppler effect is the apparent change in the frequency of waves due to:	
A a change of medium	B a change of amplitude
C relative motion between the source and the observer	D diffraction at an obstacle
Solution: It is the relative motion that alters the wavelength reaching the observer.	
MCQ 7.11.2 When the source of sound approaches a stationary observer, the observed wavelength:	
A increases	B decreases
C stays the same	D becomes zero
Solution: The same number of waves is packed into a shorter distance ahead of the source.	
MCQ 7.11.3 The apparent frequency heard when the source approaches a stationary observer is:	
A $\left(\frac{v}{v + u_s} \right) f$	B $\left(\frac{v - u_s}{v} \right) f$
C $\left(\frac{v}{v - u_s} \right) f$	D vf
Solution: Subtracting the speed of the source makes the bracket greater than one.	
MCQ 7.11.4 When the source moves away from the listener, the pitch of the sound:	
A rises	B is unchanged
C falls	D doubles
Solution: The waves are stretched out, so the apparent frequency is lower.	
MCQ 7.11.5 The change in the wavelength caused by the motion of the source is called the:	
A phase shift	B path difference
C Doppler shift	D beat frequency
Solution: It is written $\Delta\lambda = u_s/f$.	
MCQ 7.11.6 A source of 400 Hz moves towards a stationary listener at 40 m s ⁻¹ . With the speed of sound 340 m s ⁻¹ , the frequency heard is:	
A 360 Hz	B 400 Hz
C 500 Hz	D 453 Hz
Solution: The apparent frequency is $(340/300) \times 400$.	

MCQ 7.11.7 Doppler first observed the effect while studying:	
A the sound of a train	B the light emitted from a star
C waves on water	D microwaves in a cavity
Solution: He noticed that starlight differed in frequency from a similar source on the Earth.	
MCQ 7.11.8 In the derivation of the formulas of this section, the observer is taken to be:	
A moving towards the source	B moving away from the source
C stationary	D accelerating
Solution: Only the source moves; the observer stands still.	
Topic 7.12 Applications of Doppler Effect	
Scope The Doppler effect applies to electromagnetic waves as well as to sound waves, and its applications run from air traffic to medicine. Radar One of its important applications is the radar system, which uses radio waves to determine the elevation and speed of an aeroplane. RADAR, from Radio Detection And Ranging, is a device which transmits and receives radio waves. How It Works - If an aeroplane approaches the radar, the wavelength of the wave reflected from the aeroplane is <u>shorter</u>. - If the aeroplane moves away, the reflected wavelength is <u>longer</u>. The speed of satellites moving around the Earth is determined by the same principle.	
Diagram  A frequency shift is used in a radar to detect the motion of an aeroplane	
Sonar SONAR, from Sound Navigation And Ranging, is the general name for sonic or ultrasonic underwater echo-ranging and echo-sounding systems. It detects the presence of objects under water by acoustical echo. Doppler Detection It relies upon the relative speed of the target and the detector to indicate the speed of the target, using the apparent change in frequency that occurs when the source and the observer are in relative motion. Military Uses The detection and location of submarines, the control of antisubmarine weapons, mine hunting and the measurement of the depth of the sea. Astronomy Astronomers use the Doppler effect to calculate the speeds of distant stars and galaxies. The line spectrum of light from the star is compared with light from a laboratory source, the Doppler shift of the star's light is measured, and the speed of the star is then calculated. Red Shift Stars moving away from the Earth show a <u>red shift</u> . The emitted waves have a longer wavelength than if the star had been at rest, so the spectrum is shifted towards the red end. All the distant galaxies are moving away from us, and their speeds have been estimated by measuring their red shifts.	

Blue Shift

Stars moving towards the Earth show a blue shift. The wavelengths of the light emitted are shorter than if the star had been at rest, so the spectrum is shifted towards the blue end.

Diagram



Radar Speed Trap

Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off any car in the path of the microwaves, and between sending out bursts the transmitter is open to detect the reflected microwaves. If the reflection is caused by a moving obstacle, the reflected microwaves are Doppler shifted, and the speed of the car is calculated by computer programmes from the measured shift.

Satellite Navigation

The Doppler shift is used to determine the velocity and position of a satellite, which makes accurate location tracking possible.

Satellite Communication

Doppler shift compensation is applied to keep the communication signals stable.

Doppler Radar

It detects wind velocity and precipitation patterns, and the Doppler shift also helps measure the velocity and deformation of the Earth's surface.

Doppler Echocardiography

It measures blood flow velocity and detects cardiac abnormalities such as valve stenosis or regurgitation, and it is used to optimize pacemaker settings.

Doppler Ultrasound

It measures blood flow, calculates cardiac output and detects vascular stenosis or occlusion.

Book Examples

Example 7.12.1 One of the lines of the absorption spectrum of a faint galaxy is identified as the Ca-K line and is found at 478 nm. The wavelength of the same line measured in the laboratory is 397 nm. (a) Is the galaxy moving towards or away from the Earth? (b) Compute the speed of the galaxy relative to the Earth.

Given Data

$$\lambda = 397 \text{ nm} = 397 \times 10^{-9} \text{ m}$$

$$\lambda' = 478 \text{ nm} = 478 \times 10^{-9} \text{ m}$$

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

To Find

Direction of motion, and the speed $u_s = ?$

Formula

$$f = \frac{c}{\lambda}$$

$$f' = \left(\frac{c}{c + u_s} \right) f$$

Solution

The laboratory frequency of the line is

$$f = \frac{3 \times 10^8}{397 \times 10^{-9}}$$

$$= 7.56 \times 10^{14} \text{ Hz}$$

The frequency observed from the galaxy is

$$f' = \frac{3 \times 10^8}{478 \times 10^{-9}}$$

$$= 6.28 \times 10^{14} \text{ Hz}$$

The observed frequency is the lower of the two, and the line has shifted towards the red end, so the galaxy is moving away from the Earth.

Putting the values in the Doppler formula

$$3 \times 10^8 + u_s = \frac{3 \times 10^8 \times 7.56 \times 10^{14}}{6.28 \times 10^{14}}$$

$$= 3.612 \times 10^8 \text{ m s}^{-1}$$

$$u_s = 3.612 \times 10^8 - 3 \times 10^8$$

$$= 6.12 \times 10^7 \text{ m s}^{-1}$$

Result

$$u_s = 6.12 \times 10^7 \text{ m s}^{-1}$$

Short Questions

SQ 7.12.1 What does RADAR stand for, and how does it give the speed of an aeroplane?

RADAR

Radio Detection And Ranging, a device which transmits and receives radio waves.

Speed

The wave reflected from an approaching aeroplane comes back shorter in wavelength, and from a receding one longer. The size of that Doppler shift gives the speed.

SQ 7.12.2 What is meant by the red shift and the blue shift of the light from a star?

Red Shift

A star moving away from the Earth sends light of a longer wavelength than if it were at rest, so its spectrum is shifted towards the red end.

Blue Shift

A star moving towards the Earth sends light of a shorter wavelength, so its spectrum is shifted towards the blue end.

SQ 7.12.3 How does a radar speed trap measure the speed of a car?

Bursts of Microwaves

Microwaves are emitted from a transmitter in short bursts, and between the bursts the transmitter listens for the reflection.

Measurement

A moving car returns microwaves that are Doppler shifted, and a computer programme works the speed out from the size of the shift.

SQ 7.12.4 What does SONAR stand for, and give two of its military uses.

SONAR

Sound Navigation And Ranging, an underwater echo-ranging and echo-sounding system.

Military Uses

The detection and location of submarines, and mine hunting. It is also used to control antisubmarine weapons and to measure the depth of the sea.

MCQ 7.12.1 RADAR stands for:

A Radial Detection And Recording	B Radio Detection And Ranging
C Rapid Direction And Ranging	D Radio Direction And Reflection

Solution: It is a device which transmits and receives radio waves.

MCQ 7.12.2 A star moving away from the Earth shows:

- A a blue shift B no shift at all
C a doubled frequency D a red shift

Solution: Its light arrives with a longer wavelength, towards the red end of the spectrum.

MCQ 7.12.3 SONAR is used for:

- A measuring the diameters of stars B tuning musical instruments
C detecting submarines D producing beats

Solution: It is an underwater echo-ranging system, and submarine detection is among its known military uses.

MCQ 7.12.4 The waves used in a radar speed trap are:

- A microwaves B radio waves
C ultrasonic waves D X-rays

Solution: Microwaves are emitted in short bursts and the reflected bursts are examined for a Doppler shift.

MCQ 7.12.5 Doppler echocardiography measures:

- A blood flow velocity B the size of the heart
C the body temperature D the mass of the blood

Solution: The shift of the reflected ultrasound gives the speed at which the blood moves.

Chapter Exercise

SQ 7.1 What are the conditions for interference to occur?**Conditions**

The two waves must have the same frequency. They must travel in the same direction and overlap. The sources must be coherent, with a constant phase difference. Their amplitudes should be nearly equal.

SQ 7.2 Differentiate between constructive and destructive interference of waves.

Constructive Interference	Destructive Interference
A compression meets a compression.	A compression meets a rarefaction.
The displacements are added up.	The displacements cancel out.
A large displacement is seen on the CRO.	Zero displacement is seen.
$\Delta S = n\lambda$	$\Delta S = (2n + 1)\lambda/2$

SQ 7.3 What are coherent waves and coherent sources? Give examples.**Coherent Waves**

Waves of the same frequency having a constant phase difference.

Coherent Sources

Sources which emit such waves, that is, they vibrate in phase.

Examples

Two loudspeakers driven from the same audio generator.
Laser light and a monochromatic source.

SQ 7.4 Distinguish between longitudinal and transverse waves.

Longitudinal Wave	Transverse Wave
Particles vibrate <u>along</u> the direction of energy travel.	Particles vibrate at <u>right angles</u> to it.
It travels as compressions and rarefactions.	It travels as crests and troughs.
Sound waves in air.	Waves on a string and light.

SQ 7.5 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave? How is it so?**Not Directly**

A stationary wave needs two identical waves travelling in opposite directions. Waves moving the same way only interfere.

By Reflection

If the wave is reflected from the fixed end, it returns along the same path and superposes with the incident wave.

SQ 7.6 How would you apply Doppler effect in studying cardiac problems in humans?**Doppler Echocardiography**

Ultrasound is sent into the heart and the shift in frequency of the reflected waves gives the speed of the blood.

Uses

It measures blood flow velocity and cardiac output. It detects valve stenosis and regurgitation, and helps set pacemakers.

SQ 7.7 What is meant by diffraction of waves? For what purpose, the ripple tank is used?**Diffraction**

The bending of waves around the sharp edges of an obstacle, or their spreading beyond a barrier.

Ripple Tank

It generates water waves and demonstrates reflection, refraction and diffraction. A narrow gap gives more diffraction than a wide one.

CRQ 7.1 Which measurement of a wave is the most important when determining the wave's intensity?**Amplitude**

The amplitude of the wave is the most important measurement.

Reason

Energy is stored according to the displacement of the particles from the mean position. Every other factor in the intensity is fixed for a given wave in a given medium.

$$\frac{I}{A^2} = \text{constant}$$

$$I \propto A^2$$

So doubling the amplitude makes the intensity four times.

CRQ 7.2 Can you apply Doppler effect to light waves? Describe briefly.**Yes**

Doppler first observed this effect in the light of a star, whose frequency differed from a source on the Earth.

Red Shift

A star moving away gives a longer wavelength, so the spectrum shifts to the red end.

Blue Shift

A star coming closer shifts it to the blue end.

CRQ 7.3 Can you compare the compressions and rarefactions of the longitudinal wave with the peaks and troughs of the transverse wave? Discuss.**Yes**

Both are points of maximum displacement, half a wavelength apart.

Comparison

A compression is a region of high pressure and corresponds to a crest.

A rarefaction is a region of low pressure and corresponds to a trough.

Difference

Crests and troughs lie across the direction of travel, compressions and rarefactions along it.

CRQ 7.4 How should a source of sound move with respect to an observer so that the frequency of its sound does not change? Write two examples.

Condition

There must be no relative motion between the source and the observer along the line joining them.

Example 1

A source moving in a circle with the observer at its centre.

Example 2

A source and observer in the same moving car, as both have the same velocity.

CRQ 7.5 Why is it difficult to recognize beats when the frequency difference is greater than 10 Hz? Exemplify.

Reason

The number of beats per second equals the difference of the two frequencies.

$$f_b = f_1 - f_2$$

When this exceeds 10 Hz, the beats come too rapidly for the ear to separate and merge into one steady note.

Example

Forks of 320 Hz and 340 Hz give 20 beats per second, which cannot be counted.

MCQ 7.1 The simple wave speed equation is represented by:

A	$v = f\lambda$	B	$v = S/t$
C	$v = r\omega$	D	$v = \Delta d/\Delta t$

Solution: A crest travels one wavelength in one time period, so the speed is the product of frequency and wavelength.

MCQ 7.2 The principle of superposition in waves is stated as:

A	the displacement of a wave is the sum of the displacements of its individual components	B	the velocity of a wave is the product of its individual components
C	the frequency of a wave is the difference of its individual components	D	the amplitude of a wave is the ratio of its individual components

Solution: The resultant displacement of a particle acted upon by several waves is the algebraic sum of the individual displacements.

MCQ 7.3 A node in a stationary wave is:

A	a point of maximum displacement	B	a point of intermediate displacement
C	a point of zero displacement	D	a point of infinite displacement

Solution: At a node the two component waves always arrive out of phase, so the point stays permanently at rest.

MCQ 7.4 An antinode in a stationary wave is:

A	a point of maximum displacement	B	a point of minimum displacement
C	a point of zero displacement	D	a point of infinite displacement

Solution: At an antinode the component waves arrive in phase, so the amplitude is the sum of their amplitudes.

MCQ 7.5 Stationary waves are defined as:

A	waves that move with a constant velocity	B	waves that move with a changing velocity
C	waves that oscillate in a fixed position	D	waves that propagate through a medium

Solution: The pattern of nodes and antinodes stays fixed in space, so the wave neither moves forward nor carries energy past a node.

MCQ 7.6 Harmonics are:

A	integer multiples of a fundamental frequency	B	integer submultiples of a fundamental frequency
C	random frequencies	D	non-integer multiples of a fundamental frequency

Solution: The stationary waves of a system form the discrete series $f_1, 2f_1, 3f_1$ and so on.

MCQ 7.7 The result of constructive interference between two waves is represented as:

A	a decrease in amplitude	B	an increase in amplitude
C	no change in amplitude	D	a shift in phase

Solution: The two displacements are in the same direction and add up, so the resultant amplitude is larger.

MCQ 7.8 If the amplitude of the wave is tripled, then the amount of energy is increased by:

A	3 times	B	6 times
C	9 times	D	12 times

Solution: The energy of a wave is proportional to the square of its amplitude, and the square of three is nine.

MCQ 7.9 What type of waves do headphones use to produce sound?

A	Electromagnetic waves	B	Mechanical waves
C	Pressure waves	D	Longitudinal waves

Solution: The diaphragm sets the air itself vibrating, so the sound reaching the ear is carried by a material medium as a mechanical wave.

MCQ 7.10 The typical frequency range of microwaves is:

A	$10^3 - 10^5$ Hz	B	$10^5 - 10^7$ Hz
C	$10^7 - 10^9$ Hz	D	$10^8 - 10^{11}$ Hz

Solution: Microwave wavelengths run from about a metre down to a millimetre, which places their frequencies in this highest band.

MCQ 7.11 The bending of waves around an obstacle is called as:

A	refraction	B	reflection
C	diffraction	D	interference

Solution: Bending round a sharp edge, or spreading beyond a barrier, is diffraction.

MCQ 7.12 The Doppler Effect used in astronomy is for:

A	measuring the diameters of stars	B	determining velocity of galaxies
C	analyzing properties of black holes	D	studying behaviour of electromagnetic waves

Solution: The shift of the star's spectral lines against a laboratory source gives the speed at which it recedes or approaches.

Numerical 7.1 The speed of a wave on a typical string is 24 m s^{-1} . At what driving frequency will it resonate if its length is 6.0 m? (Ans: 2 Hz)

Given Data $v = 24 \text{ m s}^{-1}$
 $\ell = 6.0 \text{ m}$

To Find
Driving frequency $f_1 = ?$

Formula
The string is fixed at both ends, so the fundamental holds one loop.

$$\lambda_1 = 2\ell$$

$$f_1 = \frac{v}{2\ell}$$

Solution $f_1 = \frac{24}{2 \times 6.0} = 2 \text{ Hz}$

Result $f_1 = 2 \text{ Hz}$

Numerical 7.2 The lowest resonance frequency for a guitar string of length 0.75 m is 400 Hz. Calculate the speed of a transverse wave on the string. (Ans: 600 m s^{-1})

Given Data $\ell = 0.75 \text{ m}$
 $f_1 = 400 \text{ Hz}$

To Find
Speed of the wave $v = ?$

Formula

The lowest resonance frequency is the fundamental.

$$f_1 = \frac{v}{2\ell}$$

$$v = 2\ell f_1$$

Solution

$$v = 2 \times 0.75 \times 400 = 600 \text{ m s}^{-1}$$

Result

$$v = 600 \text{ m s}^{-1}$$

Numerical 7.3 A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded. (Ans: 314 Hz)

Given Data

$$f_A = 320 \text{ Hz}$$

$$f_b = 4 \text{ Hz}$$

$$f'_b = 6 \text{ Hz after loading}$$

To Find

Loaded frequency of B $f'_B = ?$

Formula

$$f_b = f_A - f_B$$

$$f_B = f_A \pm f_b$$

Solution

Before loading, B is either

$$f_B = 320 + 4 = 324 \text{ Hz}$$

or

$$f_B = 320 - 4 = 316 \text{ Hz}$$

Choosing Between Them

Wax lowers the frequency of B. If B were 324 Hz its frequency would move towards 320 Hz and the beats would fall below 4 per second. The beats rose to 6 instead, so B must lie below A.

Loaded Frequency

$$f'_B = 320 - 6 = 314 \text{ Hz}$$

Result

$$f'_B = 314 \text{ Hz}$$

Numerical 7.4 A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked. Density of steel is $7.8 \times 10^3 \text{ kg m}^{-3}$. (Ans: 76.2 Hz)

Given Data

$$F = 80 \text{ N}$$

$$d = 0.50 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$$

$$\ell = 1.5 \text{ m}$$

$$\rho = 7.8 \times 10^3 \text{ kg m}^{-3}$$

To Find

Fundamental frequency $f_1 = ?$

Formula

The mass per unit length of a wire of cross-section A is the density times that area.

$$m = \rho A = \rho \frac{\pi d^2}{4}$$

$$v = \sqrt{\frac{F}{m}}$$

$$f_1 = \frac{v}{2\ell}$$

Solution

The mass per unit length of the wire is

$$m = 7.8 \times 10^3 \times \frac{3.14 \times (5.0 \times 10^{-4})^2}{4}$$

$$= 1.53 \times 10^{-3} \text{ kg m}^{-1}$$

The speed of the transverse wave on the wire is

$$v = \sqrt{\frac{80}{1.53 \times 10^{-3}}} = 228.6 \text{ m s}^{-1}$$

The fundamental frequency is

$$f_1 = \frac{228.6}{2 \times 1.5} = 76.2 \text{ Hz}$$

Result

$$f_1 = 76.2 \text{ Hz}$$

Numerical 7.5 The average intensity of sunlight on the surface of the Earth is nearly 500 W m^{-2} . Determine the amount of energy that falls on a solar panel having an area of 0.50 m^2 in four hours. (Ans: $3.6 \times 10^6 \text{ J}$)

Given Data

$$I = 500 \text{ W m}^{-2}$$

$$A = 0.50 \text{ m}^2$$

$$t = 4 \text{ h} = 4 \times 3600 = 14400 \text{ s}$$

To Find

Energy received $E = ?$

Formula

$$I = \frac{E}{At}$$

$$E = IAt$$

Solution

$$E = 500 \times 0.50 \times 14400 = 3.6 \times 10^6 \text{ J}$$

Result

$$E = 3.6 \times 10^6 \text{ J}$$

Numerical 7.6 (a) If the intensity of a wave is 16 W m^{-2} and the amplitude is 2 m, what is the value of the constant k ? (b) If the intensity of a wave is 25 W m^{-2} and the constant k is 5 W m^{-4} , what is the amplitude? [Ans: (a) 4 W m^{-4} (b) 2.24 m]

Given Data

$$I_1 = 16 \text{ W m}^{-2}$$

$$A_1 = 2 \text{ m}$$

$$I_2 = 25 \text{ W m}^{-2}$$

$$k_2 = 5 \text{ W m}^{-4}$$

To Find

Constant $k_1 = ?$ and amplitude $A_2 = ?$

Formula

The intensity of a wave is proportional to the square of its amplitude.

$$I = kA^2$$

$$k = \frac{I}{A^2}$$

$$A = \sqrt{\frac{I}{k}}$$

Solution

For the first wave

$$k_1 = \frac{16}{(2)^2} = 4 \text{ W m}^{-4}$$

For the second wave

$$A_2 = \sqrt{\frac{25}{5}} = 2.24 \text{ m}$$

Result

$$k_1 = 4 \text{ W m}^{-4}$$

$$A_2 = 2.24 \text{ m}$$

Numerical 7.7 (a) A sound system produces 200 watts of power. If the sound is directed at a crowd with an area of 150 m^2 , what is the intensity of the sound? (b) A light bulb emits 100 watts of power. If the light is spread out evenly over a sphere with a surface area of 400 m^2 , what is the intensity of the light? [Ans: (a) 1.33 W m^{-2} (b) 0.25 W m^{-2}]

Given Data

$$P_1 = 200 \text{ W}$$

$$A_1 = 150 \text{ m}^2$$

$$P_2 = 100 \text{ W}$$

$$A_2 = 400 \text{ m}^2$$

To Find

Intensities $I_1 = ?$ and $I_2 = ?$

Formula $I = \frac{P}{A}$

Solution

For the sound system

$$I_1 = \frac{200}{150} = 1.33 \text{ W m}^{-2}$$

For the light bulb

$$I_2 = \frac{100}{400} = 0.25 \text{ W m}^{-2}$$

Result

$$I_1 = 1.33 \text{ W m}^{-2}$$

$$I_2 = 0.25 \text{ W m}^{-2}$$

Numerical 7.8 A radio antenna broadcasts 600 watts of power. If the signal is received at a distance of 10 km, what is the intensity of the signal? (Ans: $4 \times 10^{-7} \text{ W m}^{-2}$)

Given Data

$$P = 600 \text{ W}$$

$$r = 10 \text{ km} = 1.0 \times 10^4 \text{ m}$$

To Find

Intensity of the signal $I = ?$

Formula

The antenna radiates over the surface of a sphere of radius r

$$A = 4\pi r^2$$

$$I = \frac{P}{4\pi r^2}$$

Solution

The area over which the power is spread is

$$A = 4 \times 3.14 \times (1.0 \times 10^4)^2 = 1.256 \times 10^9 \text{ m}^2$$

The intensity at that distance is

$$I = \frac{600}{1.256 \times 10^9} = 4.8 \times 10^{-7} \text{ W m}^{-2}$$

Result $I = 4.8 \times 10^{-7} \text{ W m}^{-2}$

Note

The book prints this answer rounded to one significant figure as $4 \times 10^{-7} \text{ W m}^{-2}$.

Numerical 7.9 An organ pipe has a length of 1 m. Determine the frequencies of the fundamental and the first two harmonics above it (a) if the pipe is open at both ends and (b) if the pipe is closed at one end. (Speed of sound in air is 340 m s^{-1}) [Ans: 170 Hz, 340 Hz, 510 Hz; 85 Hz, 255 Hz, 425 Hz]

Given Data

$$\ell = 1 \text{ m}$$

$$v = 340 \text{ m s}^{-1}$$

To Find

The first three frequencies in each case

Formula

For a pipe open at both ends, with n any whole number

$$f_n = \frac{nv}{2\ell}$$

For a pipe closed at one end, with n odd only

$$f_n = \frac{nv}{4\ell}$$

Solution

Open at Both Ends

$$f_1 = \frac{340}{2 \times 1} = 170 \text{ Hz}$$

$$f_2 = 2 \times 170 = 340 \text{ Hz}$$

$$f_3 = 3 \times 170 = 510 \text{ Hz}$$

Closed at One End

$$f_1 = \frac{340}{4 \times 1} = 85 \text{ Hz}$$

The even harmonics are missing, so the next two present are the third and the fifth.

$$f_3 = 3 \times 85 = 255 \text{ Hz}$$

$$f_5 = 5 \times 85 = 425 \text{ Hz}$$

Result

170 Hz, 340 Hz, 510 Hz

85 Hz, 255 Hz, 425 Hz

Numerical 7.10 A train is approaching a station at 90 km h^{-1} , sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed? (Speed of sound is 340 m s^{-1}) (Ans: 1079.4 Hz, 931.5 Hz)

Given Data

$$u_s = 90 \text{ km h}^{-1} = 25 \text{ m s}^{-1}$$

$$f = 1000 \text{ Hz}$$

$$v = 340 \text{ m s}^{-1}$$

To Find

Apparent frequencies $f_A = ?$ on approach and $f_B = ?$ on recession

Formula

For a source approaching a stationary listener

$$f_A = \left(\frac{v}{v - u_s} \right) f$$

For a source receding from a stationary listener

$$f_B = \left(\frac{v}{v + u_s} \right) f$$

Solution

While the train approaches

$$f_A = \left(\frac{340}{340 - 25} \right) \times 1000 = 1079.4 \text{ Hz}$$

While the train moves away

$$f_B = \left(\frac{340}{340 + 25} \right) \times 1000 = 931.5 \text{ Hz}$$

Result

$$f_A = 1079.4 \text{ Hz}$$

$$f_B = 931.5 \text{ Hz}$$

Self Test

Seventeen questions across the whole chapter. Work through them without the notes, then check the answers keyed at the foot of the page.

Q 1 A wave travelling on a string is described by its speed, frequency and wavelength. If the frequency is halved while the speed stays the same, the wavelength is:

- | | | | |
|----------|---------|----------|-----------|
| A | halved | B | unchanged |
| C | doubled | D | quartered |

Q 2 Sound in air, ripples on a pond and seismic waves are all classed together as:

- | | | | |
|----------|------------------|----------|-----------------------|
| A | quantum waves | B | electromagnetic waves |
| C | transverse waves | D | mechanical waves |

Q 3 Two waves of amplitudes 6 cm and 2 cm superpose exactly out of phase. The resultant amplitude is:

- | | | | |
|----------|------|----------|------|
| A | 8 cm | B | 4 cm |
| C | 3 cm | D | zero |

Q 4 The interference of sound from two loudspeakers can be observed only because the speakers are:

- | | | | |
|----------|--------------------|----------|--------------------------------|
| A | of different sizes | B | driven from the same generator |
| C | placed far apart | D | of different frequencies |

Q 5 At a point the waves from two coherent sources differ in path by four whole wavelengths. The point is:

- | | | | |
|----------|----------------------------|----------|---------------------------|
| A | quiet | B | loud |
| C | silent only for high notes | D | unaffected by either wave |

Q 6 A stationary wave has an antinode at $x = 0$. The nearest node lies at a distance of:

- | | | | |
|----------|-------------|----------|-------------|
| A | λ | B | $\lambda/2$ |
| C | $\lambda/4$ | D | 2λ |

Q 7 A stationary wave carries no energy along the string because:

- | | | | |
|----------|---|----------|--------------------------|
| A | the two waves have different amplitudes | B | the nodes never move |
| C | the string is too short | D | the frequency is too low |

Q 8 A stretched string sounds its fundamental at 200 Hz. When it is made to vibrate in three loops, it sounds:

- | | | | |
|----------|---------|----------|---------|
| A | 66.7 Hz | B | 400 Hz |
| C | 600 Hz | D | 1800 Hz |

Q 9 A string is replaced by another of the same length and tension but four times the mass per unit length. Its fundamental frequency becomes:

- | | | | |
|----------|---------------------|----------|----------------|
| A | four times as great | B | twice as great |
| C | half as great | D | unchanged |

Q 10 An organ pipe closed at one end and one open at both ends have the same fundamental frequency. The length of the closed pipe is:

- | | | | |
|----------|--------------------------------|----------|----------------------------------|
| A | twice that of the open pipe | B | half that of the open pipe |
| C | equal to that of the open pipe | D | four times that of the open pipe |

Q 11 The third harmonic of a pipe closed at one end is 660 Hz. Its fundamental frequency is:

- | | | | |
|----------|--------|----------|--------|
| A | 110 Hz | B | 165 Hz |
| C | 220 Hz | D | 330 Hz |

Q 12 In the microwave experiment the probe detector is moved in order to locate the:

- | | | | |
|----------|-----------------|----------|---------------------|
| A | transmitter | B | nodes and antinodes |
| C | metal reflector | D | resonant cavity |

Q 13 Water waves in a ripple tank spread most on passing a gap when their wavelength is:

- | | | | |
|----------|--------------------------|----------|-------------------------|
| A | far smaller than the gap | B | comparable with the gap |
| C | zero | D | unrelated to the gap |

Q 14 A fork of 512 Hz sounded with a second fork gives 5 beats per second. After the second fork is loaded with wax the beats fall to 2 per second. The unloaded second fork was:

- | | | | |
|----------|--------|----------|--------|
| A | 507 Hz | B | 510 Hz |
| C | 514 Hz | D | 517 Hz |

Q 15 The intensity of a wave at a point 2 m from a small source is I . At 4 m from the same source it is:

- | | | | |
|----------|-------|----------|-------|
| A | $2I$ | B | $I/2$ |
| C | $I/4$ | D | $4I$ |

Q 16 A car horn sounds a steady note as the car drives past a listener. The listener hears the pitch:

- | | | | |
|----------|--------------------------|----------|---|
| A | rise steadily throughout | B | fall as the car approaches and rise as it recedes |
| C | stay the same throughout | D | drop as the car passes |

Q 17 An astronomer finds the spectral lines of a galaxy shifted towards longer wavelengths. The galaxy is:

- | | | | |
|----------|-------------------------------|----------|-----------------------------|
| A | moving towards the Earth | B | moving away from the Earth |
| C | at rest relative to the Earth | D | rotating about its own axis |

Answers 1 (c) 2 (d) 3 (b) 4 (b) 5 (b) 6 (c) 7 (b) 8 (c) 9 (c) 10 (b) 11 (c) 12 (b) 13 (b) 14 (d) 15 (c) 16 (d) 17 (b)