

TOPIC 1 Oscillatory Motion

SQ 14.1.1 Define oscillatory motion.

Definition

Oscillatory motion is a to-and-fro motion about a mean position. It is a periodic motion which repeats itself after an equal interval of time.

SQ 14.1.2 Give any four examples of oscillatory motion.

Examples

A simple pendulum vibrating about its mean position.
A mass oscillating at the end of a spring.
The atoms or molecules in a solid oscillating about their mean position.
The motion of a swing.

SQ 14.1.3 How do sound waves and musical instruments show oscillatory motion?

Answer

Air molecules oscillate when sound waves travel through air. Most musical instruments like the guitar and violin have strings attached to them to produce music by vibratory motion.

SQ 14.1.4 What causes oscillatory motion of a body?

Answer

All bodies undergoing oscillatory motion have a mean position. When the body is displaced from this position there is a restoring force which brings it back to equilibrium, and this causes the oscillatory motion.

SQ 14.1.5 Define one oscillation.

Definition

One complete round trip or cycle of a vibrating body about its mean position is called an oscillation.

Alternative

It is the motion of the body from one extreme position to the other extreme position and back to the first, crossing the mean position.

SQ 14.1.6 Define instantaneous displacement.

Definition

The distance of a vibrating body from its mean position at any instant is called instantaneous displacement, denoted by x .

SQ 14.1.7 Define amplitude.

Definition

The magnitude of the maximum displacement of the vibrating body on either side of its mean position is called amplitude, denoted by x_0 .

SQ 14.1.8 Define time period and write its unit.

Definition

Time period is the time taken to complete one vibration or one cycle. It is represented by T and its SI unit is the second.

SQ 14.1.9 Write time period in terms of frequency and angular frequency.

Formula

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

SQ 14.1.10 Define frequency and write its unit and dimension.

Definition

Frequency is the number of vibrations or oscillations completed by a vibrating body per unit time.

Unit

Its unit is the hertz, where one hertz is one oscillation per second, and its dimension is $[T^{-1}]$.

SQ 14.1.11 Define angular frequency and write its unit.

Definition

Angular displacement per unit time is called angular frequency.

Formula

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f$$

Unit

Its SI unit is rad s^{-1} .

SQ 14.1.12 A fish connected to a spring makes 10 vibrations in 20 seconds. Find its period and frequency.

Solution

$$f = \frac{10}{20} = 0.5 \text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{0.5}$$

Result

$$T = 2 \text{ s}$$

SQ 14.1.13 If x_0 is the amplitude of a simple pendulum, what distance is covered in one cycle?

Answer

In one complete vibration the bob moves from one extreme to the other and back, so the distance covered is $4x_0$.

TOPIC 2 Simple Harmonic Motion

SQ 14.2.1 Define simple harmonic motion.

Definition

Simple harmonic motion is a type of oscillation or vibratory motion produced under the action of a restoring force.

SQ 14.2.2 State the necessary conditions for simple harmonic motion.

Conditions

The restoring force shall be directly proportional to the displacement from the mean position.

The force and displacement should follow Hooke's law,
 $F = -kx$

The acceleration should be proportional to the displacement,
 $a \propto -x$.

SQ 14.2.3 What does the negative sign in $a \propto -x$ indicate?**Answer**

The negative sign indicates that the acceleration is always directed towards the mean position.

SQ 14.2.4 Are all periodic vibrations examples of simple harmonic motion?**Answer**

No. All restoring forces are not always proportional to the displacement. Any restoring force can cause oscillatory motion, but only a force proportional to displacement gives simple harmonic motion.

SQ 14.2.5 Why is the motion of an electrocardiogram needle not simple harmonic?**Reason**

An electrocardiogram traces the periodic pattern of a beating heart, but the restoring force in this case is not always proportional to the displacement from the mean position.

SQ 14.2.6 On what does the spring constant depend?**Answer**

The spring constant k depends upon the nature of the material of the spring, that is its physical shape and structure.

TOPIC 3 Practical S.H.M Systems**SQ 14.3.1 Derive the condition for S.H.M for a mass attached to a spring.****Derivation**

Comparing

$$F = ma$$

with Hooke's law

$$F = -kx$$

gives

$$ma = -kx \quad \text{so} \quad a = -\frac{k}{m}x.$$

Result

Since $\frac{k}{m}$ is constant, $a \propto -x$, which is the mathematical form of S.H.M.

SQ 14.3.2 What does the relation $a \propto -x$ show for a mass-spring system?**Answer**

It shows that the acceleration of a body executing S.H.M is directly proportional to the displacement and is always directed towards the mean position.

SQ 14.3.3 Write the frequency and time period of a mass-spring system.**Formulas**

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

SQ 14.3.4 Give real world applications that utilize the S.H.M principle.**Applications**

Pendulum clocks, musical instruments and vehicle suspension systems.

SQ 14.3.5 A particle in S.H.M has $\omega = 2 \text{ rad s}^{-1}$. Find its frequency and time period.**Solution**

$$f = \frac{\omega}{2\pi} = \frac{2}{2\pi}$$

Result

$$f = 0.318 \text{ Hz} \quad T = 3.14 \text{ s}$$

SQ 14.3.6 What is a simple pendulum?**Definition**

A simple pendulum consists of a small heavy bob of mass m suspended from a rigid support through a light inextensible string of certain length.

SQ 14.3.7 Which component of weight provides the restoring force in a simple pendulum?**Answer**

The component $mg \cos \theta$ and the tension are along the same line but opposite, so they cancel each other. The component $mg \sin \theta$ provides the necessary restoring force towards the mean position.

SQ 14.3.8 Derive the acceleration of a simple pendulum.**Derivation**

The restoring force is

$$F = -mg \sin \theta$$

and by Newton's second law

$$ma = -mg \sin \theta$$

Result

$$a = -g \sin \theta$$

SQ 14.3.9 Why is $\sin \theta$ replaced by θ for a simple pendulum?**Reason**

For small values of θ

$\sin \theta$ is nearly equal to θ when measured in radian. Hence

$$a = -g\theta$$

SQ 14.3.10 Show that the motion of a simple pendulum is S.H.M.**Derivation**

Since

$$\theta = \frac{S}{l}$$

and for a small angle the arc S is nearly equal to the displacement x , therefore

$$a = -\frac{g}{l}x.$$

Result

If l and g are constant, then $a \propto -x$, which is the mathematical form of S.H.M.

SQ 14.3.11 Write the time period of a simple pendulum.**Formula**

$$T = 2\pi \sqrt{\frac{l}{g}}$$

SQ 14.3.12 On what does the time period of a simple pendulum depend?

Answer

The time period is directly proportional to the square root of the length of the string and inversely proportional to the square root of g .

Independence

It is independent of the mass of the bob and of the amplitude.

SQ 14.3.13 What is a second pendulum?

Definition

A pendulum that completes one vibration in two seconds is known as a second pendulum.

SQ 14.3.14 How is a simple pendulum used to measure g ?

Method

The length l of the pendulum is measured and the pendulum is set into motion. The time period T is measured using a stopwatch.

Formula

$$g = \frac{4\pi^2 l}{T^2}$$

SQ 14.3.15 Find the length of a second pendulum where $g = 9.8 \text{ m s}^{-2}$.

Solution

Using

$$l = \frac{gT^2}{4\pi^2}$$

with

$$T = 2 \text{ s}$$

Calculation

$$l = \frac{9.8 \times 4}{4\pi^2}$$

Result

$$l = 0.99 \text{ m}$$

SQ 14.3.16 Will the time period of a pendulum change if shifted from Lahore to Karachi?

Answer

Yes. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

the time period depends on g . As the value of g differs slightly between the two places, the time period will change slightly.

SQ 14.3.17 Would the time period of a pendulum be the same on the Earth and the Moon?

Answer

No. The value of g on the Moon is much smaller than on the Earth, so for the same length the time period on the Moon would be greater.

SQ 14.3.18 Why is a small amplitude recommended while measuring the time period of a pendulum?

Reason

The derivation uses the approximation $\sin \theta \approx \theta$, which is valid only for small angles. For large amplitudes this approximation fails and the motion is no longer simple harmonic.

SQ 14.3.19 Why does a vibrating simple pendulum not produce sound?

Reason

The frequency of a simple pendulum is far below the audible range of frequencies, so the vibrations it produces in the air cannot be heard as sound.

SQ 14.3.20 Who patented the first pendulum clock and when?

Answer

The Dutch mathematician, astronomer and physicist Christiaan Huygens patented the first pendulum clock in 1656. He finalized the mathematical formula that relates pendulum length to time using simple harmonic motion.

TOPIC 4 Simple Harmonic Motion and Uniform Circular Motion

SQ 14.4.1 Why is S.H.M correlated with uniform circular motion?

Reason

The different parameters such as displacement, velocity, acceleration and time period of S.H.M can be understood by relating it with uniform circular motion.

Observation

When a particle moves in a circular path, its projection on a diameter executes S.H.M.

SQ 14.4.2 Describe the turntable experiment demonstrating S.H.M.

Experiment

A turntable of radius x_0 has a ball attached to its rim, and a beam of light casts a shadow of the ball on a screen.

Observation

When the turntable rotates with constant angular speed, the shadow oscillates to and fro across the screen in the form of simple harmonic motion.

SQ 14.4.3 Write the instantaneous displacement of a body executing S.H.M.

Formula

$$x = x_0 \cos \omega t$$

SQ 14.4.4 Derive the instantaneous velocity of a body executing S.H.M.

Derivation

The velocity of the projection is the horizontal component of v_P , so

$$v = v_P \cos(90^\circ - \theta) = v_P \sin \theta$$

Since

$$v_P = x_0 \omega$$

Result

$$v = x_0 \omega \sin \omega t$$

SQ 14.4.5 Write the velocity of a body in S.H.M in terms of displacement.

Formula

$$v = \omega \sqrt{x_0^2 - x^2}$$

SQ 14.4.6 Write the acceleration of a body executing S.H.M.

Formula

$$a = -\omega^2 x$$

SQ 14.4.7 A 0.2 kg mass on a spring of $k = 10 \text{ N m}^{-1}$ is displaced 0.1 m. Find its angular frequency.

Solution

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{0.2}}$$

Result

$$\omega = 7.07 \text{ rad s}^{-1}$$

SQ 14.4.8 For the above system, find the acceleration at $x = 0.05 \text{ m}$.

Solution

$$a = -\omega^2 x = -(7.07)^2 \times 0.05$$

Result

$$a = -2.5 \text{ m s}^{-2}$$

SQ 14.4.9 For the above system, find the velocity at $x = 0.05 \text{ m}$.

Solution

$$v = \omega \sqrt{x_0^2 - x^2} = 7.07 \sqrt{(0.1)^2 - (0.05)^2}$$

Result

$$v \approx 0.61 \text{ m s}^{-1}$$

TOPIC 5 Phase

SQ 14.5.1 Define phase.

Definition

The angle

$$\theta = \omega t$$

which specifies the displacement as well as the direction of motion of a point oscillating in S.H.M is called phase.

SQ 14.5.2 What does phase show?

Answer

Phase is the quantity which shows the state of motion of an oscillator.

SQ 14.5.3 Write the general equation of S.H.M including the phase constant.

Formula

$$x = x_0 \cos(\omega t + \phi)$$

Here the time varying quantity $(\omega t + \phi)$ is called the phase of the motion.

SQ 14.5.4 What is the phase constant and on what does it depend?

Definition

The constant ϕ is called the phase constant or phase angle.

Dependence

Its value depends on the displacement and velocity of the particle at $t = 0$

SQ 14.5.5 What does the quantity ϕ represent between two oscillators?

Answer

The quantity ϕ represents the phase difference between the states of motion of two oscillators.

SQ 14.5.6 Describe the motion for $\phi = 0$.

Answer

At

$$t = 0$$

$$\frac{T}{2}$$

and T , corresponding to

$$\theta = 0$$

π and 2π , the point is at the extreme positions.

At

$$t = \frac{T}{4}$$

and $\frac{3T}{4}$, the point is at the mean position.

SQ 14.5.7 Describe the motion for $\phi = \pi/2$.

Answer

At

$$t = 0$$

$$\frac{T}{2}$$

and T the point is at the mean position.

At

$$t = \frac{T}{4}$$

and $\frac{3T}{4}$ the point is at the extreme position.

SQ 14.5.8 When are two oscillating systems said to be out of phase?

Answer

When the phase difference between two oscillating systems is π , they are said to be oscillating out of phase.

SQ 14.5.9 When are two oscillating systems said to be in phase?

Answer

When the phase difference between the oscillating systems is 0 or 2π , they are said to be oscillating in phase.

TOPIC 6 Graphical Representation of S.H.M

SQ 14.6.1 Describe the displacement graph of a body executing S.H.M.

Answer

The displacement is given by

$$x = x_0 \cos \omega t$$

This shows that the particle has maximum displacement x_0 at the extreme position.

SQ 14.6.2 Describe the velocity graph of a body executing S.H.M.

Answer

The velocity is given by

$$v = -x_0 \omega \sin \omega t$$

The velocity is maximum, equal to $x_0 \omega$, at the mean position and zero at the extreme positions.

SQ 14.6.3 Describe the acceleration graph of a body executing S.H.M.

Answer

The acceleration is given by

$$a = -x_0\omega^2 \cos \omega t$$

It is maximum, equal to $x_0\omega^2$, at the extreme positions and zero at the mean position.

SQ 14.6.4 What is the phase difference between velocity and displacement in S.H.M?

Answer

The phase difference between velocity and displacement is $\frac{\pi}{2}$.

SQ 14.6.5 What is the phase difference between acceleration and displacement in S.H.M?

Answer

The phase difference between acceleration and displacement is π .

TOPIC 7 Conservation of Energy in S.H.M

SQ 14.7.1 Which energies does a body executing S.H.M possess?

Answer

It possesses potential energy on account of its displacement from the mean position, and kinetic energy due to its velocity.

SQ 14.7.2 What happens to the total energy during S.H.M?

Answer

These energies vary during the oscillation, but the total energy at any instant remains constant in the absence of unbalanced frictional forces.

SQ 14.7.3 What is the average restoring force from the mean to the extreme position?

Derivation

The force is zero at the mean position and maximum, kx_0 , at the extreme position.

Result

$$F_{av} = \frac{0 + kx_0}{2} = \frac{1}{2}kx_0$$

SQ 14.7.4 Derive the work done in stretching a spring to its maximum displacement.

Derivation

The work done is the average force multiplied by the displacement, so

$$W = \frac{1}{2}kx_0 \times x_0.$$

Result

$$W = \frac{1}{2}kx_0^2$$

SQ 14.7.5 What is elastic potential energy?

Definition

The work done on the mass attached to a spring is stored in terms of potential energy, called elastic potential energy.

SQ 14.7.6 Write the instantaneous potential energy of an oscillator.

Formula

$$(PE)_{inst} = \frac{1}{2}kx^2$$

SQ 14.7.7 Where is the potential energy of an oscillator zero and where is it maximum?

Answer

The potential energy is zero at

$$x = 0$$

and maximum at

$$x = \pm x_0$$

that is at the extreme positions where the total energy is entirely elastic potential energy.

Formula

$$(PE)_{max} = \frac{1}{2}kx_0^2$$

SQ 14.7.8 Write the instantaneous kinetic energy of an oscillator.

Formula

$$(KE)_{inst} = \frac{1}{2}k(x_0^2 - x^2)$$

SQ 14.7.9 Where does the mass attain maximum velocity and what is the maximum kinetic energy?

Answer

At the mean position

$$x = 0$$

the mass gets maximum velocity.

Formula

$$(KE)_{max} = \frac{1}{2}kx_0^2$$

SQ 14.7.10 Write the total energy of a body executing S.H.M.

Formula

$$E_T = (KE) + (PE) = \frac{1}{2}kx_0^2$$

This remains constant throughout the motion.

SQ 14.7.11 A 0.2 kg mass needs 40 N to stretch it 0.1 m. Find the spring constant.

Solution

$$k = \frac{F}{x} = \frac{40}{0.1}$$

Result

$$k = 400 \text{ N m}^{-1}$$

SQ 14.7.12 For $k = 400 \text{ N m}^{-1}$ and amplitude 0.1 m, find the total energy.

Solution

$$E_T = \frac{1}{2}kx_0^2 = \frac{1}{2}(400)(0.1)^2$$

Result

$$E_T = 2 \text{ J}$$

SQ 14.7.13 For the above oscillator, find the kinetic energy at $x = 0.03 \text{ m}$.

Solution

$$KE = \frac{1}{2}k(x_0^2 - x^2) = 200 [(0.1)^2 - (0.03)^2]$$

Result

$$KE = 1.82 \text{ J}$$

SQ 14.7.14 For the above oscillator, find the potential energy when K.E. equals P.E.

Solution

When

$$KE = PE$$

each is half of the total energy, so

$$PE = \frac{E_T}{2} = \frac{2}{2}.$$

Result

$$PE = 1 \text{ J}$$

SQ 14.7.15 For the above oscillator, find the maximum acceleration.

Solution

$$a = \frac{kx_0}{m} = \frac{400 \times 0.1}{0.2}$$

Result

$$a = 200 \text{ m s}^{-2}$$

TOPIC 8 Free and Forced Oscillations

SQ 14.8.1 Define free vibrations.

Definition

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

SQ 14.8.2 Give an example of free vibrations.

Example

A simple pendulum vibrates freely with its natural frequency, which depends only upon its length, when it is slightly displaced from its mean position.

SQ 14.8.3 What happens to the total energy in free oscillations?

Answer

In free oscillations the total energy of the body remains constant, that is energy is conserved. In the absence of resistive force the amplitude of the oscillation remains constant.

SQ 14.8.4 Define forced vibrations.

Definition

If a freely oscillating system is subjected to an external force, then forced vibrations take place.

Example

When a swing is struck repeatedly, forced vibrations are produced.

SQ 14.8.5 Give two examples of forced vibrations.

Examples

The vibrations of a factory floor caused by the running of heavy machinery.

The loud music produced by the sounding wooden boards of string instruments.

TOPIC 9 Damped Oscillation

SQ 14.9.1 Define damped oscillations and damping forces.

Definition

The oscillations with decreasing amplitude in the presence of various resistive forces are called damped oscillations, and the resistive forces are called damping forces.

SQ 14.9.2 Why does an oscillating body eventually come to rest in practice?

Reason

In practical life there are dissipating forces such as air resistance and friction. The amplitude of the oscillation gradually decreases with time due to these forces and finally the body comes to rest.

SQ 14.9.3 Give examples of damped oscillation.

Examples

A girl swinging on a swing, where damping occurs due to gravity and air friction.

The amplitude of an oscillating simple pendulum decreasing gradually with time.

Touching an oscillating tuning fork with our finger.

SQ 14.9.4 Define light damping and give an example.

Definition

Light damping reduces gradually the energy and amplitude of a vibrating body.

Example

A swing in a playground is the best example of light damping.

SQ 14.9.5 Define heavy damping and give an example.

Definition

Heavy damping is a type of damping which causes the system to return to equilibrium very slowly without oscillating.

Example

A pendulum submerged in thick oil experiences a heavy resistance from the oil.

SQ 14.9.6 Define critical damping and give an example.

Definition

Critical damping is a type of damping in which the object returns to the equilibrium position in the shortest possible time.

Example

Shock absorbers in a car.

SQ 14.9.7 Give an application of damped oscillation.

Application

Shock absorbers used in the suspension system of a car are a practical application of damped oscillation.

Purpose

The damping system ensures a comfortable ride for passengers when the car is moving on a bumpy, rough road, by damping the excessive oscillations.

TOPIC 10 Resonance

SQ 14.10.1 Why can a damped oscillator not maintain its natural frequency for long?

Reason

Due to the resistive forces, the amplitude of the oscillation decreases gradually with time.

Remedy

We can maintain constant amplitude by applying a periodic external force which is called a driving force.

SQ 14.10.2 Describe the experiment demonstrating resonance with pendulums.

Setup

Two pairs of pendulums A and B of length l_1 , and C and D of length l_2 , are suspended by a horizontal string. Another pendulum P of length l is introduced.

SQ 14.10.3 What is observed in the pendulum resonance experiment?

Observation

When P has length l_1 and is set into vibration, pendulums A and B receive a driving force through the string and their amplitude increases, because their natural frequency is the same as that of P .

Result

Pendulums C and D , whose natural frequencies differ, continue to remain at rest.

SQ 14.10.4 How is food cooked in a microwave oven by resonance?

Explanation

The waves produced have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by the water and fat molecules in the food, heating them up.

SQ 14.10.5 Why do plastic or glass containers not heat up in a microwave oven?

Reason

Only food containing water molecules can be heated by the microwave oven. Plastic or glass containers do not heat up since they do not contain water molecules.

SQ 14.10.6 How does radio tuning work on the principle of resonance?

Explanation

We change the natural frequency of the electrical circuit of the receiver to make it equal to the transmission frequency of the radio station.

Result

When the two frequencies match, resonance occurs, the energy absorbed is maximum, and this is the only station we hear.

SQ 14.10.7 Why are marching soldiers advised to break step while crossing a bridge?

Reason

If the soldiers march in step, the frequency of their footsteps may become equal to the natural frequency of the bridge. The bridge may then be set into vibrations with large amplitude due to resonance and may collapse.

SQ 14.10.8 Why can resonance be dangerous for an aeroplane wing?

Reason

The wings experience forces such as aerodynamic force, turbulence and engine vibration. When the frequencies of these forces match the natural frequency of the wings, resonance occurs and it may be dangerous.

SQ 14.10.9 How does the sound board of a musical instrument work by resonance?

Explanation

When a musician strikes the string of a guitar or violin, the string produces sound waves. These vibrations transfer their energy to the hollow wooden box of the instrument.

Result

The sound box resonates, amplifying the sound waves and producing a louder and richer sound.

SQ 14.10.10 How is resonance used in magnetic resonance imaging?

Explanation

Magnetic resonance imaging is a medical diagnostic technique in which strong radio frequency radiations are used to cause nuclei to oscillate, and energy is absorbed by the nuclei.

Result

The patterns of the energy absorbed are used to produce a computer enhanced photograph.

SQ 14.10.11 How is the amplitude of a swing increased?

Answer

The amplitude of a swing can be increased by applying a suitable periodic force on it regularly. The natural frequency of the swing must match the frequency of the pushes, amplifying the motion.

SQ 14.10.12 How does resonance relate to earthquakes?

Answer

Resonance is the oscillation up and down or back and forth motion caused by seismic waves. During an earthquake buildings oscillate, and damage is severe when the frequency of the seismic waves matches their natural frequency.

TOPIC 11 Sharpness of Resonance

SQ 14.11.1 On what do the amplitude and sharpness of resonance depend?

Answer

The amplitude at resonance, as well as its sharpness, both depend upon the damping.

SQ 14.11.2 What is the effect of smaller damping on resonance?

Answer

The smaller the damping, the greater will be the amplitude and the more sharp will be the resonance.

SQ 14.11.3 What does the resonance curve of a heavily damped system look like?

Answer

A heavily damped system has a fairly flat resonance curve on an amplitude-frequency graph.