

Topic 16.1 Electric Potential Difference**Electrostatics****Definition**

Electrostatics or static electricity refers to the phenomena arising from stationary electric charges. These include the forces exerted by charges on each other, the electric field and electric potential produced by them, and the electrical energy stored in space and in various devices.

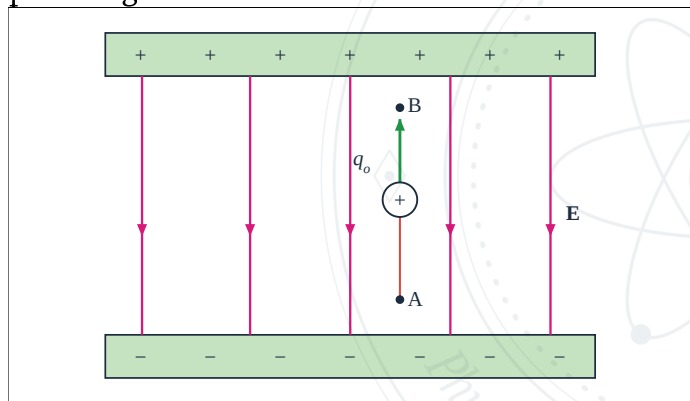
Coulomb's Law

A charge placed in the electric field of another charge experiences a force. Its direction depends on whether the charges are positive or negative.

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Electric Potential Difference**Definition**

The electric potential difference is the work done per unit charge in moving it between two points against the electric field.



$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0} = \frac{\Delta U}{q_0}$$

Unit

Its SI unit is the volt. It can be converted into other units such as kilovolts (kV) or millivolts (mV).

$$1 \text{ V} = \frac{1 \text{ J}}{1 \text{ C}}$$

Potential Gradient

The electric field represents the rate of change of potential with distance, called the gradient of potential.

$$\Delta V = -\mathbf{E} \cdot \Delta \mathbf{d}$$

$$E = -\frac{\Delta V}{\Delta d}$$

The negative sign indicates that the field points from higher to lower potential.

Uniform Field

In a uniform electric field the relation simplifies to

$$E = \frac{V}{d}$$

The potential difference between two points is directly proportional to the electric field

strength and the distance between them. A stronger electric field causes a more rapid decrease in potential, a clear physical link between field intensity and voltage variation in space.

For Your Information

Although the points at infinity are assumed to be at zero potential, the Earth is taken to be at zero potential practically. The absolute value of the electric potential at a point is therefore measured with respect to the Earth.

Book Examples

Example 16.1.1 A 6 volts battery is connected to two parallel copper plates separated by a distance of 0.3 cm. Find the electric field intensity between them.

Given Data

$$\Delta V = 6 \text{ V}$$

$$\Delta d = 0.3 \text{ cm} = 0.003 \text{ m}$$

To Find

Electric field intensity $E = ?$

Formula $E = \frac{\Delta V}{\Delta d}$

Solution

Putting the values in the formula

$$E = \frac{6 \text{ V}}{0.003 \text{ m}} = 2000 \text{ V m}^{-1}$$

Result

$$E = 2000 \text{ V m}^{-1} = 2000 \text{ N C}^{-1}$$

Short Questions

SQ 16.1.1 Define electric potential difference and give its SI unit.

Potential difference

The work done per unit charge in moving it between two points against the electric field.

$$\Delta V = \frac{W_{AB}}{q_0}$$

Its SI unit is the volt, one joule per coulomb.

SQ 16.1.2 What is meant by the gradient of potential?

Gradient of potential

The rate of change of potential with distance.

The electric field is equal to the negative of the potential gradient.

$$E = -\frac{\Delta V}{\Delta d}$$

SQ 16.1.3 Show that volt per metre is the same as newton per coulomb.

Equivalent units

$$1 \text{ V m}^{-1} = 1 \text{ J C}^{-1} \text{ m}^{-1} = 1 \text{ N m C}^{-1} \text{ m}^{-1} = 1 \text{ N C}^{-1}$$

Both are units of the electric field intensity.

MCQ 16.1.1 The SI unit of electric potential difference is the:

- A coulomb B farad
C **volt** D joule

Solution: One volt is one joule per coulomb.**MCQ 16.1.2 A 12 V battery is connected to two parallel plates 0.4 cm apart. The field between them is:**

- A **3000 V m⁻¹** B 30 V m⁻¹
C 48 V m⁻¹ D 300 V m⁻¹

Solution: $E = V/d = 12 \text{ V} / 0.004 \text{ m} = 3000 \text{ V m}^{-1}$.**MCQ 16.1.3 One volt is equal to:**

- A 1 C J⁻¹ B **1 J C⁻¹**
C 1 J s⁻¹ D 1 N C⁻¹

Solution: Potential difference is work per unit charge.**MCQ 16.1.4 If the separation of two plates is doubled at the same potential difference, the uniform field between them becomes:**

- A double B four times
C unchanged D **half**

Solution: $E = V/d$, so E halves when d doubles.**Topic 16.2 Electric Potential****Electric Potential****Definition**

The electric potential at any point in an electric field is equal to the work done in bringing a unit positive charge from infinity to that point keeping it in electrostatic equilibrium.

$$V = \frac{W}{q_0}$$

Taking A at infinity with zero potential, the potential at B is the work done in bringing q_0 from infinity to B, with the subscripts dropped.

Reference Point

To give a concept of electric potential at a point, a reference must be assigned zero electric potential. This point is usually taken at infinity.

Scalar

The potential at a point is still a potential difference, between that point and infinity. Both potential and potential difference are scalar quantities, because both W and q_0 are scalars.

LQ 1 starts here — an expression for the absolute electric potential at a point due to a point charge q

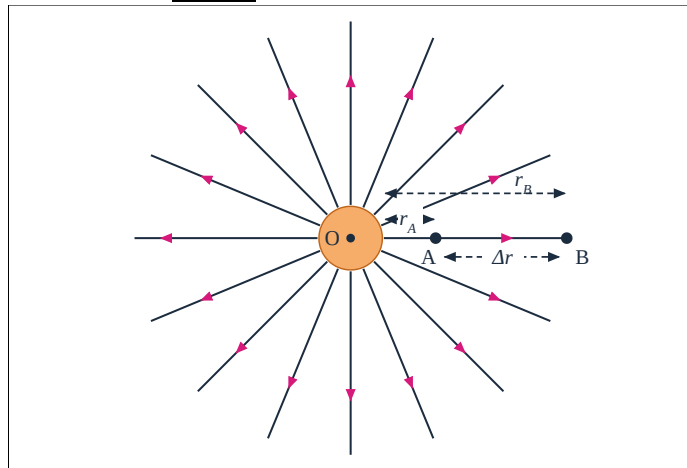
Potential due to a Point Charge**Statement**

The absolute electric potential at a point near a point charge q is directly proportional to the charge and inversely proportional to its distance r from the charge.

Set Up

A unit positive charge is brought from infinity to the point keeping it in electrostatic equilibrium. The relation of a uniform field needs E constant, but E varies inversely as the square of the distance from the point charge, so

the potential is computed from basic principles. The field is radial.

**Two Close Points**

Take two points A and B infinitesimally close to each other, so that E remains almost constant between them. Their distances from q are r_A and r_B , and the mid-point of the interval is at distance r

$$r_B = r_A + \Delta r$$

$$\Delta r = r_B - r_A$$

$$r = \frac{r_A + r_B}{2}$$

Field at the Mid-point

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

As A and B are very close, the arithmetic mean can be taken equal to the geometric mean as a first approximation.

$$r^2 = r_A r_B$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A r_B}$$

Potential Difference

If a unit positive charge is moved from B to A, the work done equals the potential difference between A and B.

$$V_A - V_B = -E(r_A - r_B)$$

$$V_A - V_B = E(r_B - r_A)$$

Substituting the value of E

$$V_A - V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{r_B - r_A}{r_A r_B} \right) q$$

$$V_A - V_B = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

Absolute Potential

To find the potential at A, the point B is taken at infinity, so that V_B is zero.

$$\frac{1}{r_B} = \frac{1}{r_\infty} = \frac{1}{\infty} = 0$$

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A}$$

General Expression

The electric potential V_r at a distance r from q is

$$V_r = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} \right)$$

Sign

The electric potential is negative if the charge q is negative.

Several Charges

Where there is more than one charge, each contributes, and the total electric potential at a point is the algebraic sum of the individual potentials.

LQ 1 ends here

Do You Know

Bug-zappers, with their characteristic bluish glow, attract certain insects and electrocute them with a very high voltage.

For Your Information

The energy used to operate an electric car comes from the energy stored in its battery pack. The voltage of the pack and the magnitude of the charge determine how much energy is consumed from the battery.

Book Examples

Example 16.2.1 Find the electric potential at a point 1.2 m from a charge of 5.0×10^{-8} C.

Given Data $q = 5.0 \times 10^{-8}$ C
 $r = 1.2$ m

To Find
Electric potential $V = ?$

Formula $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

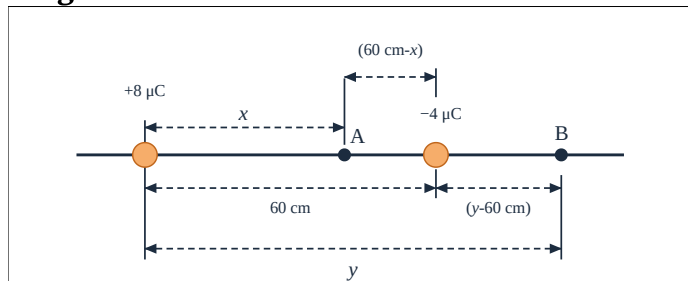
Solution
Putting the values in the formula
$$V = \frac{(9 \times 10^9 \text{ N m}^2 \text{ C}^{-2})(5.0 \times 10^{-8} \text{ C})}{1.2 \text{ m}}$$
$$= 375 \text{ V}$$

Result $V = 375 \text{ V}$

Example 16.2.2 Two point charges $+8 \mu\text{C}$ and $-4 \mu\text{C}$ are placed 60 cm apart as shown in Fig. 16.3. Where is the electric potential zero on the line passing through the charges?

Given Data
 $q_1 = +8 \mu\text{C} = 8 \times 10^{-6} \text{ C}$
 $q_2 = -4 \mu\text{C} = -4 \times 10^{-6} \text{ C}$
Separation between the charges
60 cm = 0.6 m

Diagram



To Find
The points of zero electric potential

Formula

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

The potential is zero where the potentials of the two charges are equal in magnitude. The positive charge is the larger, so there are two such points, both closer to the smaller negative charge: A between the charges and B outside the negative charge.

Solution

Let A lie at a distance x from the positive charge. Balancing the two potentials at A

$$\frac{1}{4\pi\epsilon_0} \left(\frac{8 \times 10^{-6} \text{ C}}{x} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{4 \times 10^{-6} \text{ C}}{0.6 \text{ m} - x} \right)$$

$$4.8 \text{ m} - 8x = 4x$$

$$12x = 4.8 \text{ m}$$

$$x = 0.4 \text{ m} = 40 \text{ cm}$$

Let B lie at a distance y from the positive charge. Balancing the two potentials at B

$$\frac{1}{4\pi\epsilon_0} \left(\frac{4 \times 10^{-6} \text{ C}}{y - 0.6 \text{ m}} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{8 \times 10^{-6} \text{ C}}{y} \right)$$

$$4y = 8(y - 0.6 \text{ m})$$

$$y = 2y - 1.2 \text{ m}$$

$$y = 1.2 \text{ m} = 120 \text{ cm}$$

Result

The potential is zero at A, 40 cm from the positive charge between the charges, and at B, 120 cm from the positive charge beyond the negative charge.

Short Questions

SQ 16.2.1 Define electric potential at a point.

Electric potential

The work done in bringing a unit positive charge from infinity to the point, keeping it in electrostatic equilibrium.

$$V = \frac{W}{q_0}$$

SQ 16.2.2 Why is electric potential a scalar quantity?

Ratio of two scalars

It is the work done per unit charge, and both the work W and the charge q_0 are scalars. Potentials of several charges therefore add algebraically.

SQ 16.2.3 Why is the arithmetic mean of r_A and r_B replaced by their geometric mean in finding the potential of a point charge?

Points very close

A and B are infinitesimally close, so the two means are almost equal. The geometric mean gives the field in terms of the product of the two distances.

$$r^2 = r_A r_B$$

SQ 16.2.4 How is the electric potential at a point due to several charges found?

Algebraic sum

Each charge contributes its own potential, positive for a positive charge and negative for a negative one, and the total is their algebraic sum.

$$V_r = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

MCQ 16.2.1 The electric potential at a distance r from a point charge q is:

- | | |
|--|--|
| A $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ | B $\frac{1}{4\pi\epsilon_0} \frac{q^2}{r}$ |
| C $4\pi\epsilon_0 qr$ | D $\frac{1}{4\pi\epsilon_0} \frac{q}{r}$ |

Solution: The potential falls off as $1/r$, the field as $1/r^2$.

MCQ 16.2.2 The electric potential at 3 m from a charge of 2×10^{-9} C is:

- | | |
|-------|--------|
| A 6 V | B 18 V |
| C 2 V | D 54 V |

Solution: $V = 9 \times 10^9 \times 2 \times 10^{-9} / 3 = 6$ V.

MCQ 16.2.3 If the distance from a point charge is doubled, the electric potential becomes:

- | | |
|---------------|-------------|
| A one quarter | B half |
| C double | D unchanged |

Solution: V is inversely proportional to r .

MCQ 16.2.4 The point of zero electric potential is usually taken at:

- | | |
|---------------------|--------------------------------|
| A the charge itself | B the surface of the charge |
| C infinity | D the mid-point of two charges |

Solution: Practically, the Earth is taken at zero potential.

MCQ 16.2.5 The potential at the mid-point between charges $+q$ and $-q$ is:

- | | |
|-------------------------------------|-------------------------------|
| A twice the potential of $+q$ alone | B equal to that of $+q$ alone |
| C infinite | D zero |

Solution: The two equal and opposite potentials add algebraically to zero.

Topic 16.3 Electric Potential Energy

Motion of a Charge in a Field

Positive Charge

A charge q_o placed in the field of a charge q and allowed to move freely is repelled by q . The potential decreases with distance, so a free positive charge always accelerates from a point of higher potential to lower potential.

Gain in K.E.

If the potential difference between the points is ΔV , the potential energy of the charge decreases by $q_o \Delta V$, and this decrease appears as its kinetic energy. A body of mass m carrying q_o reaches the point of lower potential energy with velocity v

$$\frac{1}{2}mv^2 = q_o \Delta V$$

Negative Charge

A negative charge allowed to move freely in an electric field moves from a point of lower potential to higher potential.

Electron Volt

Energy can be expressed as the product of potential difference and charge. The electron volt is the energy acquired or lost by an electron accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = \text{charge on an electron} \times 1 \text{ volt}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

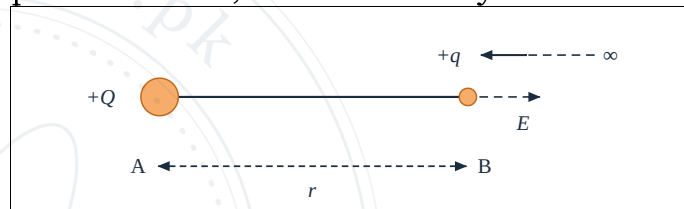
Potential Energy of Two Point Charges

Definition

The electric potential energy of a system of two point charges is the work done in bringing one charge from infinity to its position in the field of the other.

Potential at B

The concept of electric potential leads directly to the electric potential energy of two point charges. The charge Q at A creates an electric potential V at B, a distance r away.



$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Work in Bringing q

If another charge q is brought from infinity to B, work has to be done against the electric field.

$$W_B = q \times (\text{Electric potential})$$

$$W_B = qV$$

Putting the value of V

$$W_B = q \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r} \right)$$

$$W_B = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

Total Work

The total work done in placing the two charges at A and B, dropping the subscript, is

$$W = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

Potential Energy

By definition, this work is stored as the electric potential energy U of the system.

$$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

Book Examples

Example 16.3.1 Two point charges are separated by a distance of 4.0 cm. If the charges are 8 μC and 5 μC , what is the electric potential energy of the two charges?

Given Data

$$r = 4 \times 10^{-2} \text{ m}$$

$$Q = 8 \mu\text{C} = 8 \times 10^{-6} \text{ C}$$

$$q = 5 \mu\text{C} = 5 \times 10^{-6} \text{ C}$$

To Find

Electric potential energy $U = ?$

Formula $U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$

Solution

Putting the values in the formula

$$U = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \left[\frac{(8 \times 10^{-6} \text{ C})(5 \times 10^{-6} \text{ C})}{4 \times 10^{-2} \text{ m}} \right]$$

$$= 9 \text{ J}$$

Result $U = 9 \text{ J}$

Short Questions

SQ 16.3.1 What happens to the potential energy of a positive charge that moves freely through a potential difference?

It decreases, and the decrease appears as kinetic energy.

Energy conservation

The charge loses potential energy $q_o \Delta V$ and gains the same kinetic energy.

$$\frac{1}{2}mv^2 = q_o \Delta V$$

SQ 16.3.2 What is the work done in bringing a charge q from infinity to a point at distance r from a charge Q ?

Work against the field

The work is the charge times the potential of Q at that point.

$$W_B = qV$$

$$W_B = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

SQ 16.3.3 What does a negative electric potential energy of two charges show?

Unlike charges

The charges are of opposite sign, so they attract. Work must be done on the system to separate them to infinity.

$$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

SQ 16.3.4 How does the potential energy of two like charges change when they are brought closer?

It increases.

Inverse dependence on r

$$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

$$\frac{Qq}{4\pi\epsilon_0} = \text{constant}$$

$$U \propto \frac{1}{r}$$

Halving the separation doubles the potential energy.

MCQ 16.3.1 An electron accelerated through a potential difference of 50 V gains:

- | | |
|-----------------------------------|---------|
| A 0.5 eV | B 50 eV |
| C $3.2 \times 10^{-21} \text{ J}$ | D 50 J |

Solution: Energy gained = $e \times 50 \text{ V} = 50 \text{ eV} = 8 \times 10^{-18} \text{ J}$.

MCQ 16.3.2 One electron volt is equal to:

- | | |
|-----------------------------------|-----------------------------------|
| A $1.6 \times 10^{-19} \text{ C}$ | B $6.25 \times 10^{18} \text{ J}$ |
| C $1.6 \times 10^{-19} \text{ J}$ | D 1 J |

Solution: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V}$.

MCQ 16.3.3 If the distance between two point charges is halved, their electric potential energy:

- | | |
|----------------------|--------------------|
| A doubles | B halves |
| C becomes four times | D remains the same |

Solution: U is inversely proportional to r .

MCQ 16.3.4 A proton is accelerated through 1000 V. The kinetic energy it gains is:

- | | |
|-----------------------------------|-----------------------------------|
| A $1.6 \times 10^{-22} \text{ J}$ | B $1.6 \times 10^{-16} \text{ J}$ |
| C 1000 J | D $6.25 \times 10^{21} \text{ J}$ |

Solution: K.E. = $q\Delta V = 1.6 \times 10^{-19} \times 1000 = 1.6 \times 10^{-16} \text{ J}$.

MCQ 16.3.5 The electric potential energy of +2 μC and -3 μC placed 0.3 m apart is:

- | | |
|------------|-----------|
| A -0.18 J | B +0.18 J |
| C -0.054 J | D -1.8 J |

Solution: $U = 9 \times 10^9 \times (2 \times 10^{-6})(-3 \times 10^{-6})/0.3 = -0.18 \text{ J}$.

Topic 16.4 Capacitors

LQ 2 starts here — what a capacitor is, the capacitance of a parallel plate capacitor and the role of a dielectric

Capacitor

Definition

A capacitor is a device that can store electrical energy in the form of an electric field. Two types are discussed.

- Isolated spherical conductors
- Parallel plate capacitors

Isolated Spherical Conductor

Definition

A spherical conductor placed in space such that it is far from other conductors or influence is known as an isolated spherical conductor.

Capacitance

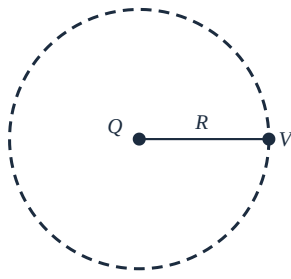
The ability of an isolated hollow spherical conductor to store electric charge is called its capacitance C . It is defined as the electric charge stored on the sphere per unit electric potential at its surface.

$$C = \frac{Q}{V}$$

Symbols

- Q is the charge stored on the sphere.

- V is the electric potential created at the surface of the sphere by Q .



Unit

The SI unit of capacitance is the farad (F). For a spherical conductor, 1 farad is the capacitance of a sphere that stores 1 coulomb of charge when its electric potential is 1 volt, assuming it is isolated in free space.

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

Capacitance of a Sphere

The electric potential V at the surface of a sphere of radius R is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

Substituting it in the definition of capacitance

$$C = \frac{Q}{\left(\frac{1}{4\pi\epsilon_0} \frac{Q}{R}\right)}$$

$$C_{vac} = 4\pi\epsilon_0 R$$

In a Medium

In a medium of dielectric constant ϵ_r

$$C_{med} = 4\pi\epsilon_0\epsilon_r R$$

Factors

The capacitance of a spherical conductor is independent of the charge or potential. It depends on the radius of the sphere and the medium. It increases with the radius, so larger spheres store more charge at the same surface potential, and a dielectric medium also increases it.

Short Questions

SQ 16.4.1 What is a capacitor?

Capacitor

A device that can store electrical energy in the form of an electric field, and with it electric charge.

SQ 16.4.2 What is an isolated spherical conductor?

Isolated spherical conductor

A spherical conductor placed in space far from other conductors or their influence.

SQ 16.4.3 On what factors does the capacitance of a spherical conductor depend?

Radius and medium

It depends on the radius of the sphere and the medium around it, not on its charge or potential.

$$C_{med} = 4\pi\epsilon_0\epsilon_r R$$

SQ 16.4.4 Why does a larger sphere store more charge at the same surface potential?

Capacitance grows with radius

$$C_{vac} = 4\pi\epsilon_0 R$$

$$4\pi\epsilon_0 = \text{constant}$$

$$C \propto R$$

A larger radius gives a larger capacitance, so more charge for the same potential.

MCQ 16.4.1 The capacitance of an isolated sphere of radius R in vacuum is:

- | | |
|------------------------------|------------------------|
| A $\frac{R}{4\pi\epsilon_0}$ | B $4\pi\epsilon_0 R^2$ |
| C $4\pi\epsilon_0 R$ | D $\epsilon_0 R$ |

Solution: $C = Q/V$ with $V = Q/(4\pi\epsilon_0 R)$.

MCQ 16.4.2 The capacitance of an isolated sphere of radius 9 m in vacuum is about:

- | | |
|---------|--------|
| A 1 F | B 9 nF |
| C 81 pF | D 1 nF |

Solution: $C = 4\pi\epsilon_0 R = 9/(9 \times 10^9) = 1 \times 10^{-9} \text{ F}$.

MCQ 16.4.3 The capacitance of a spherical conductor depends on its:

- | | |
|--------------------------------|------------------------|
| A charge | B potential |
| C radius and the medium | D charge and potential |

Solution: It is independent of the charge or the potential.

MCQ 16.4.4 The capacitance of a sphere of radius R in a medium of dielectric constant ϵ_r is:

- | | |
|---|--------------------------------|
| A $\frac{4\pi\epsilon_0 R}{\epsilon_r}$ | B $4\pi\epsilon_0 R$ |
| C $\frac{\epsilon_r R}{4\pi\epsilon_0}$ | D $4\pi\epsilon_0\epsilon_r R$ |

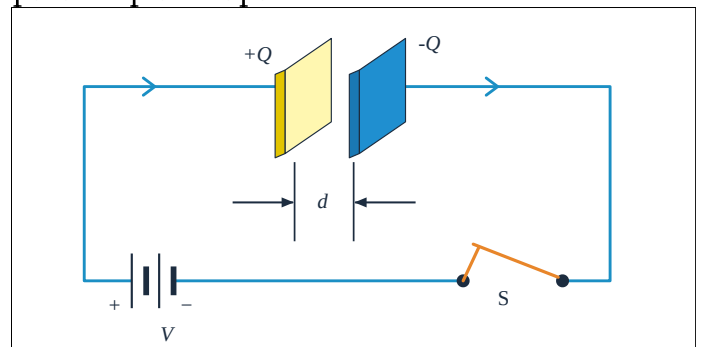
Solution: A dielectric medium increases the capacitance by the factor ϵ_r .

Topic 16.5 Parallel Plate Capacitors

Parallel Plate Capacitor

Definition

A capacitor consists of two conductors placed near one another, separated by vacuum, air or any other insulator, known as a dielectric. When the conductors are parallel plates it is a parallel plate capacitor.



Charging

When the plates are connected to a battery of voltage V , it places a charge $+Q$ on the plate connected with its positive terminal and $-Q$ on the other plate. The charge is found to be proportional to the potential difference.

$$Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V}$$

Capacitance

The proportionality constant C is the capacitance. It depends upon the geometry of the plates and the medium between them, and measures the ability of the capacitor to store charge. It is the amount of charge on one plate necessary to raise the potential of that plate by one volt with respect to the other.

Capacitance of a Parallel Plate Capacitor

Statement

The capacitance of a parallel plate capacitor is directly proportional to the area of the plates and inversely proportional to the separation between them, and it is enhanced by a dielectric between them.

Set Up

Two parallel metal plates, each of area A , are separated by a small distance d , so the electric field E between them is uniform and confined almost entirely between the plates. With air or vacuum between them

$$C_{vac} = \frac{Q}{V}$$

Field between the Plates

The field is related to the potential difference and the separation.

$$E = \frac{V}{d}$$

By Gauss's law the field between the plates depends on the surface charge density σ

$$E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \frac{Q}{A}$$

$$E = \frac{Q}{A\epsilon_0}$$

Result

Equating the two expressions for E

$$\frac{V}{d} = \frac{Q}{A\epsilon_0}$$

$$\frac{Q}{V} = \frac{A\epsilon_0}{d}$$

$$C_{vac} = \frac{A\epsilon_0}{d}$$

Role of a Dielectric

Definition

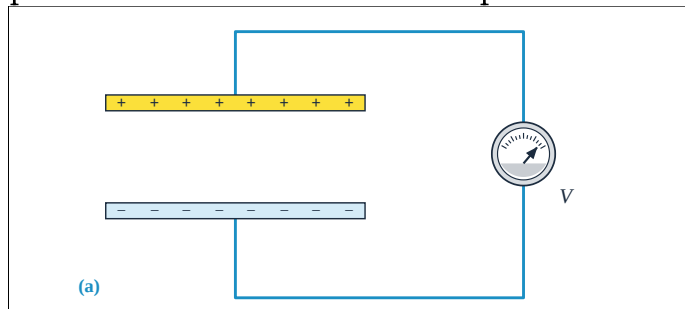
A dielectric is an insulating material, with no free electrons, placed between the plates of a capacitor.

Effect on Capacitance

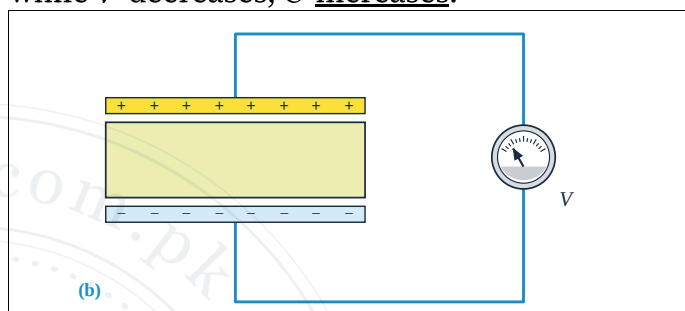
If an insulating material, a dielectric of relative permittivity ϵ_r , is introduced between the plates, the capacitance is enhanced by the factor ϵ_r , which is also called the dielectric constant.

Experiment

A charged capacitor has its plates connected to a voltmeter, whose deflection measures the potential difference between the plates.



When a dielectric is inserted between the plates the reading drops, showing a decrease in the potential difference. Since Q remains constant while V decreases, C increases.



$$C_{med} = \frac{A\epsilon_0\epsilon_r}{d}$$

The capacitance thus depends on the area of the plates, the separation between them and the medium between them.

Dielectric Constant

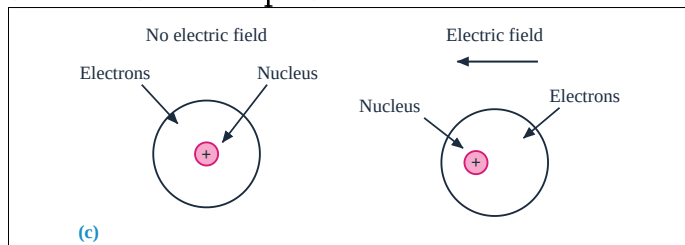
Dividing the two capacitances

$$\epsilon_r = \frac{C_{med}}{C_{vac}}$$

The dielectric constant is the ratio of the capacitance of a parallel plate capacitor with an insulating substance between the plates to its capacitance with vacuum (or air) between them.

Polarization

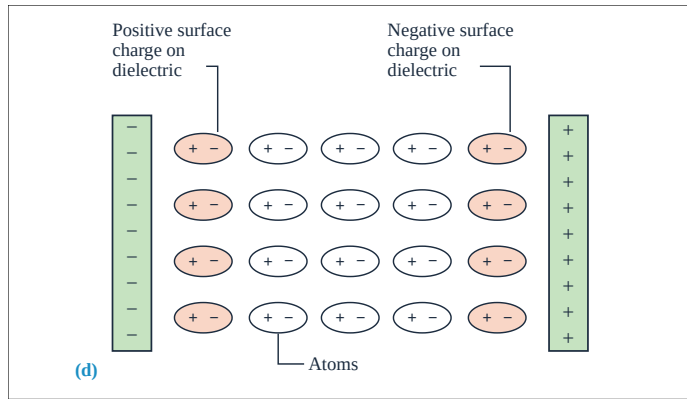
A dielectric is an insulator with no free electrons; all its electrons are bound to their nuclei. In an electric field the electrons of an atom are displaced slightly relative to the nucleus, opposite to the field. This is polarization of the dielectric, and the distorted atom is said to be polarized.



Decrease in Potential Difference

In a slab placed between the charged plates, the atoms are polarized by the field between the plates. Charges appear on the surfaces of the slab in contact with the plates, of polarity opposite to the adjacent plate. This effectively decreases the charge on the plates, so the

electric intensity and the potential difference between them decrease.



LQ 2 ends here

Short Questions

SQ 16.5.1 What is a dielectric?

Dielectric

An insulator placed between the plates of a capacitor. It has no free electrons: all its electrons are bound to their nuclei.

SQ 16.5.2 Write the capacitance of a parallel plate capacitor with a dielectric and name the factors it depends on.

Capacitance with a dielectric

$$C_{med} = \frac{A\epsilon_o\epsilon_r}{d}$$

It depends on the area of the plates, the separation between them and the medium between them.

SQ 16.5.3 What is polarization of a dielectric?

Polarization

In an electric field the electrons of each atom are displaced slightly relative to the nucleus, opposite to the field. The distorted atom is said to be polarized.

SQ 16.5.4 What is surface charge density? How is the field between the plates related to it?

Surface charge density

The charge per unit area of a plate. By Gauss's law the field between the plates is proportional to it.

$$\sigma = \frac{Q}{A}$$

$$E = \frac{\sigma}{\epsilon_o}$$

SQ 16.5.5 Why is the field between the plates of a parallel plate capacitor uniform?

Small separation

The distance d is small compared with the plates, so the field is confined almost entirely between them and is the same everywhere there.

$$E = \frac{V}{d}$$

MCQ 16.5.1 The capacitance of a parallel plate capacitor with vacuum between its plates is:

- | | |
|----------------------------------|-----------------------------------|
| A $\frac{d\epsilon_o}{A}$ | B $\frac{A\epsilon_o}{d}$ |
| C $A\epsilon_o d$ | D $\frac{A}{\epsilon_o d}$ |

Solution: From $V/d = Q/(A\epsilon_o)$.

MCQ 16.5.2 If the plate area of a parallel plate capacitor is doubled and the separation halved, its capacitance becomes:

- | | |
|---------------------|--------------------|
| A four times | B unchanged |
| C double | D half |

Solution: C is proportional to A/d , which becomes $2/(1/2) = 4$ times.

MCQ 16.5.3 The dielectric constant of vacuum is:

- | | |
|-------------------|---------------------------------|
| A 1 | B zero |
| C infinite | D 8.85×10^{-12} |

Solution: $\epsilon_r = C_{med}/C_{vac} = 1$ for vacuum.

MCQ 16.5.4 An air capacitor of $2 \mu\text{F}$ is filled with a dielectric of dielectric constant 5. Its capacitance becomes:

- | | |
|----------------------------|--------------------------|
| A $0.4 \mu\text{F}$ | B $2 \mu\text{F}$ |
| C $10 \mu\text{F}$ | D $7 \mu\text{F}$ |

Solution: $C_{med} = \epsilon_r C_{vac} = 5 \times 2 \mu\text{F}$.

MCQ 16.5.5 When a dielectric is placed in an electric field, the electrons of its atoms are displaced:

- | | |
|--------------------------|---------------------------------------|
| A along the field | B perpendicular to the field |
| C not at all | D opposite to the field |

Solution: The negative electrons are pulled against the field relative to the nucleus.

MCQ 16.5.6 A parallel plate air capacitor has plates of area 0.1 m^2 1 mm apart. Its capacitance is:

- | | |
|---|---|
| A $8.85 \times 10^{-13} \text{ F}$ | B $8.85 \times 10^{-10} \text{ F}$ |
| C $8.85 \times 10^{-8} \text{ F}$ | D $8.85 \times 10^{-11} \text{ F}$ |

Solution: $C = 8.85 \times 10^{-12} \times 0.1 / 10^{-3} = 8.85 \times 10^{-10} \text{ F}$.

Topic 16.6 Combined Capacitance of Capacitors

Combination of Capacitors

Definition

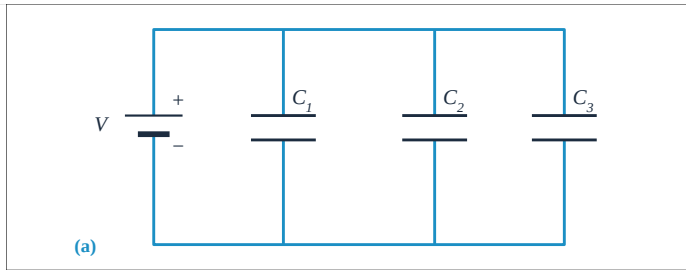
When two or more capacitors are connected together in a circuit, it is known as a combination of capacitors. It is done to obtain a desired capacitance: the combination behaves as a single equivalent capacitor, whose capacitance is the combined capacitance.

- Parallel combination
- Series combination

Parallel Combination

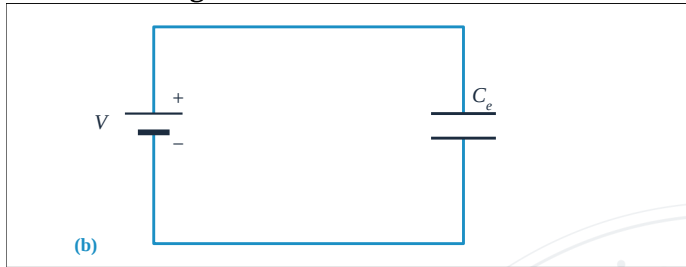
Definition

In a parallel combination the plates of one side of all the capacitors are connected to the negative terminal and the plates of the other side to the positive terminal of the battery.



Same Potential Difference

The potential difference V across each capacitor is the same, while the charge on each depends on its capacitance. The charge Q on the equivalent capacitor C_e equals what the three hold together under the same V .



$$Q = Q_1 + Q_2 + Q_3$$

Putting the charge of each capacitor

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

$$Q_3 = C_3 V$$

$$Q = C_e V$$

$$C_e V = C_1 V + C_2 V + C_3 V$$

$$C_e = C_1 + C_2 + C_3$$

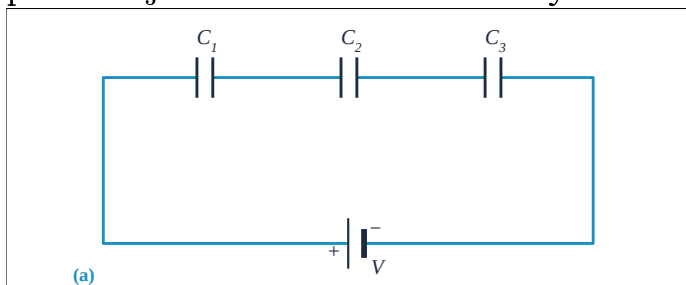
When capacitors are connected in parallel, their combined capacitance is equal to the sum of their individual capacitances.

LQ 3 starts here — the formula for the series combination of capacitors

Series Combination

Definition

In a series combination the capacitors are connected one after the other in a line. The second plate of C_1 is connected to the first plate of C_2 , and the second plate of C_2 to the first plate of C_3 . The first plate of C_1 and the second plate of C_3 are connected to the battery.



Same Charge

The battery sends a charge $+Q$ to the first plate of C_1 . Its second plate is insulated from the first and draws no charge from the battery, but $+Q$ induces an equal charge $-Q$ on it. The first plate of C_2 , connected to it, gets $+Q$, and so on,

so every capacitor receives a charge of magnitude Q on each plate.

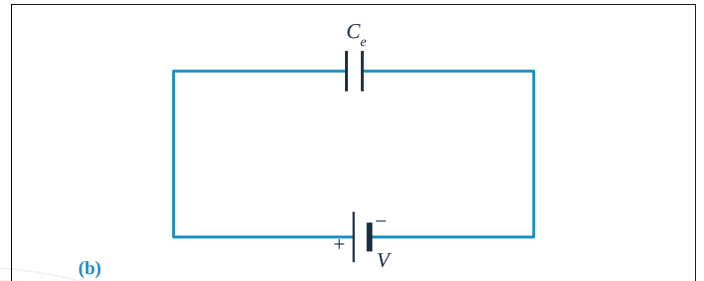
Different Potential Differences

The potential difference across the combination equals that of the battery V . The potential difference across each capacitor depends on its capacitance, and they add up to V

$$V = V_1 + V_2 + V_3$$

Equivalent Capacitance

The equivalent capacitor C_e carries the same charge Q with the potential difference V across it.



Using the formula for potential difference for each capacitor

$$V = \frac{Q}{C_e}$$

$$V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

$$V_3 = \frac{Q}{C_3}$$

Putting these values in the sum of the potential differences

$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances of the capacitors connected in series.

LQ 3 ends here

Book Examples

Example 16.6.1 A capacitor of 80 pF capacitance is charged to potential difference of 48 volts. Its plates are then connected in parallel to another capacitor. The potential difference across the combination is found to be 30 volts. What is the capacitance of the second capacitor?

Given Data

$$C_1 = 80 \text{ pF} = 80 \times 10^{-12} \text{ F}$$

$$V = 48 \text{ V}$$

$$V_2 = 30 \text{ V}$$

To Find

Capacitance of the second capacitor $C_2 = ?$

Formula $Q = CV$

$$C_2 = \frac{Q_2}{V_2}$$

Solution

Before connecting, the charge on C_1 is

$$Q_1 = C_1 V = 80 \times 10^{-12} \text{ F} \times 48 \text{ V} \\ = 3840 \times 10^{-12} \text{ C}$$

After connecting C_2 in parallel, the charge on C_1 is

$$Q'_1 = C_1 V_2 = 80 \times 10^{-12} \text{ F} \times 30 \text{ V} \\ = 2400 \times 10^{-12} \text{ C}$$

The total charge remains constant, so the charge shifted to C_2 is

$$Q_2 = Q_1 - Q'_1 = 3840 \times 10^{-12} \text{ C} - 2400 \times 10^{-12} \text{ C} \\ = 1440 \times 10^{-12} \text{ C}$$

$$C_2 = \frac{1440 \times 10^{-12} \text{ C}}{30 \text{ V}} \\ = 48 \times 10^{-12} \text{ F}$$

Result $C_2 = 48 \text{ pF}$

Example 16.6.2 Two capacitors of capacitances $12 \mu\text{F}$ and $4 \mu\text{F}$ are connected in series to a 80 V battery. (i) What is the total charge stored in the combination? (ii) What is the potential difference across each capacitor?

Given Data

$$C_1 = 12 \mu\text{F} = 12 \times 10^{-6} \text{ F}$$

$$C_2 = 4 \mu\text{F} = 4 \times 10^{-6} \text{ F}$$

$$V = 80 \text{ V}$$

To Find

(i) Total charge $Q = ?$

(ii) Potential differences $V_1 = ?$ and $V_2 = ?$

Formula

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q = C_e V$$

$$V_1 = \frac{Q}{C_1}$$

Solution

The equivalent capacitance is

$$\frac{1}{C_e} = \frac{1}{12 \times 10^{-6} \text{ F}} + \frac{1}{4 \times 10^{-6} \text{ F}} = \frac{1+3}{12 \times 10^{-6} \text{ F}} \\ = \frac{1}{3 \times 10^{-6} \text{ F}}$$

$$C_e = 3 \times 10^{-6} \text{ F}$$

(i) The total charge is

$$Q = C_e V = 3 \times 10^{-6} \text{ F} \times 80 \text{ V} \\ = 2.4 \times 10^{-4} \text{ C}$$

(ii) In series each capacitor carries the same charge Q

$$V_1 = \frac{2.4 \times 10^{-4} \text{ C}}{12 \times 10^{-6} \text{ F}} \\ = 20 \text{ V}$$

$$V_2 = \frac{2.4 \times 10^{-4} \text{ C}}{4 \times 10^{-6} \text{ F}} \\ = 60 \text{ V}$$

Result

$$Q = 2.4 \times 10^{-4} \text{ C}$$

$$V_1 = 20 \text{ V}$$

$$V_2 = 60 \text{ V}$$

Short Questions

SQ 16.6.1 Why are capacitors combined?

Combination of capacitors

To obtain a desired capacitance. The combination behaves as a single equivalent capacitor.

SQ 16.6.2 Which quantity is the same for every capacitor in a parallel combination, and which in a series combination?

Common quantity

In parallel the potential difference across each capacitor is the same; in series the charge on each capacitor is the same.

SQ 16.6.3 Why is the equivalent capacitance of a series combination less than the smallest capacitance?

Sum of reciprocals

The reciprocal of C_e is the sum of the reciprocals, so it is larger than any one of them, and C_e is smaller than the smallest.

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

SQ 16.6.4 How does every capacitor in series receive the same charge?

Charge by induction

The charge $+Q$ on the first plate induces $-Q$ on the second plate of C_1 , which leaves $+Q$ on the first plate of C_2 joined to it, and so on along the line.

MCQ 16.6.1 Three capacitors of $6 \mu\text{F}$ each are connected in parallel. The equivalent capacitance is:

A $18 \mu\text{F}$	B $2 \mu\text{F}$
C $6 \mu\text{F}$	D $9 \mu\text{F}$

Solution: $C_e = C_1 + C_2 + C_3$.

MCQ 16.6.2 Three capacitors of $6 \mu\text{F}$ each are connected in series. The equivalent capacitance is:

A $18 \mu\text{F}$	B $6 \mu\text{F}$
C $2 \mu\text{F}$	D $3 \mu\text{F}$

Solution: $1/C_e = 3/6$, so $C_e = 2 \mu\text{F}$.

MCQ 16.6.3 Capacitors of $2 \mu\text{F}$ and $3 \mu\text{F}$ are connected in series. The equivalent capacitance is:

A $5 \mu\text{F}$	B $6 \mu\text{F}$
C $0.83 \mu\text{F}$	D $1.2 \mu\text{F}$

Solution: $C_e = (2 \times 3)/(2 + 3) = 1.2 \mu\text{F}$.

MCQ 16.6.4 In a series combination of capacitors, the charge on each capacitor is:

- | | |
|-----------------------------------|---|
| A proportional to its capacitance | B the same |
| C zero | D inversely proportional to its capacitance |

Solution: Each plate gets its charge by induction from the one before.

MCQ 16.6.5 In a parallel combination of capacitors, the potential difference across each capacitor is:

- | | |
|---|-------------------|
| A proportional to its capacitance | B the same |
| C inversely proportional to its capacitance | D zero |

Solution: All are connected directly across the same battery.

MCQ 16.6.6 Capacitors of 4 μF and 12 μF in series are connected to 16 V. The potential difference across the 4 μF capacitor is:

- | | |
|--------|---------------|
| A 4 V | B 8 V |
| C 16 V | D 12 V |

Solution: $C_e = 3 \mu\text{F}$, $Q = 48 \mu\text{C}$, and $V = 48 \mu\text{C} / 4 \mu\text{F} = 12 \text{ V}$.

Topic 16.7 Energy Stored in a Capacitor

LQ 4 starts here — the electric potential energy stored in a capacitor from the area under the potential-charge graph

Energy Stored in a Capacitor

Definition

The energy stored in a capacitor is the work done in charging it, and it is stored in the electric field between its plates.

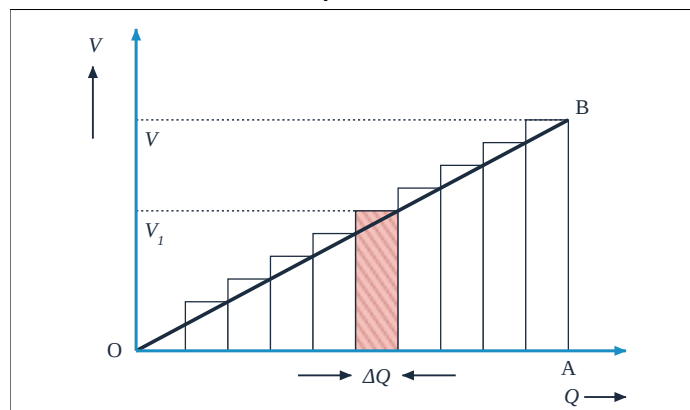
Stored Energy

A capacitor can be thought of as a device for storing electrical energy. If the plates of a charged capacitor are joined by a thick wire, charge flows from one plate to the other and the wire becomes hot. The energy that appears as heat was possessed by the charged capacitor.

V-Q Graph

The charge Q on the plates increases with the potential V between them, so a graph of V against Q is a straight line through the origin.

$$Q \propto V$$



Work for a Small Charge

The area under the graph is divided into very small increments of charge ΔQ . The work

required to move a small charge ΔQ through a potential difference V_1 across the plates is

$$\Delta W = (\Delta Q)(V_1)$$

This is the area of the rectangle shown red in the figure.

Total Work

The total work done to charge the capacitor to its final charge Q is the sum of the areas of all such rectangles, which is the total area under the graph, the area of the right angled triangle OAB.

$$\text{Area of triangle OAB} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(OA)(AB)$$

With OA equal to the total charge Q and AB to the final potential difference V

$$\text{Area of the triangle OAB} = \frac{1}{2}QV$$

Energy Stored

$$W = \frac{1}{2}QV$$

In Terms of Capacitance

$$Q = CV$$

$$W = \frac{1}{2}(CV)V$$

$$W = \frac{1}{2}CV^2$$

$$P.E. = \frac{1}{2}CV^2$$

Energy Density

Definition

Energy density is the energy stored per unit volume of the electric field.

Energy in the Field

The energy is stored in the electric field between the plates, rather than as the potential energy of the charges on the plates.

Substituting the potential difference and the capacitance with a dielectric

$$V = Ed$$

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$\text{Energy} = \frac{1}{2} \left(\frac{A\epsilon_0\epsilon_r}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 \epsilon_r E^2 \times (Ad)$$

Here Ad is the volume between the plates.

Energy Density

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

This equation is valid for any electric field strength.

LQ 4 ends here

For Your Information

A device called a defibrillator uses the charge stored in a capacitor to deliver a controlled electric shock that can restore normal heart rhythm.

Book Examples

Example 16.7.1 Two capacitors of capacitances $4\ \mu\text{F}$ and $8\ \mu\text{F}$ are connected in series and charged by a $120\ \text{V}$ power supply. Find the total charge and energy stored in the system.

Given Data

$$C_1 = 4\ \mu\text{F} = 4 \times 10^{-6}\ \text{F}$$

$$C_2 = 8\ \mu\text{F} = 8 \times 10^{-6}\ \text{F}$$

$$V = 120\ \text{V}$$

To Find

Total charge $Q = ?$ and energy stored $W = ?$

Formula

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q = C_e V$$

$$W = \frac{1}{2} QV$$

Solution

The equivalent capacitance is

$$\frac{1}{C_e} = \frac{1}{4 \times 10^{-6}\ \text{F}} + \frac{1}{8 \times 10^{-6}\ \text{F}}$$

$$= \frac{3}{8 \times 10^{-6}\ \text{F}}$$

$$C_e = 2.67 \times 10^{-6}\ \text{F}$$

The total charge is

$$Q = C_e V = (2.67 \times 10^{-6}\ \text{F}) \times 120\ \text{V}$$

$$= 3.2 \times 10^{-4}\ \text{C}$$

The energy stored in the system is

$$W = \frac{1}{2} (3.2 \times 10^{-4}\ \text{C}) \times 120\ \text{V}$$

$$= 1.92 \times 10^{-2}\ \text{J}$$

Result

$$Q = 3.2 \times 10^{-4}\ \text{C}$$

$$W = 1.92 \times 10^{-2}\ \text{J}$$

Example 16.7.2 A $16\ \mu\text{F}$ capacitor is charged to a potential difference of $100\ \text{V}$. Determine the energy stored in it. If the potential difference is doubled by what factor does the energy increase?

Given Data

$$C = 16\ \mu\text{F} = 16 \times 10^{-6}\ \text{F}$$

$$V = 100\ \text{V}$$

To Find

Energy stored $W = ?$ and the factor when V is doubled

Formula $W = \frac{1}{2} CV^2$

Solution

Putting the values in the formula

$$W = \frac{1}{2} (16 \times 10^{-6}\ \text{F}) \times (100\ \text{V})^2$$

$$= 8 \times 10^{-2}\ \text{J}$$

The energy depends on V^2 , so with V doubled

$$W' = \frac{1}{2} C(2V)^2 = 4 \times \frac{1}{2} CV^2 = 4W$$

Result

$$W = 8 \times 10^{-2}\ \text{J}$$

The energy increases by a factor of 4.

Short Questions

SQ 16.7.1 What is the evidence that a charged capacitor possesses energy?

Heating on discharge

If the plates are joined by a thick wire, charge flows and the wire becomes hot. The heat came from the energy of the charged capacitor.

SQ 16.7.2 What is the energy density of an electric field?

Energy density

The energy stored per unit volume of the field. It holds for any electric field strength.

$$\text{Energy density} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

SQ 16.7.3 Why is the energy of a charged capacitor one half of QV and not QV ?

Rising potential

While charging, the potential rises from zero to V , so the work is the area of the triangle under the V - Q graph, not of a rectangle.

$$W = \frac{1}{2} QV$$

MCQ 16.7.1 The energy stored in a capacitor of charge Q at potential difference V is:

A $\frac{1}{2} QV$

B QV

C $2QV$

D $\frac{Q}{V}$

Solution: It is the area of the triangle under the V - Q graph.

MCQ 16.7.2 The energy stored in a $2\ \mu\text{F}$ capacitor charged to $100\ \text{V}$ is:

A $0.02\ \text{J}$

B $0.1\ \text{J}$

C $0.01\ \text{J}$

D $200\ \text{J}$

Solution: $W = \frac{1}{2} \times 2 \times 10^{-6} \times 100^2 = 0.01\ \text{J}$.

MCQ 16.7.3 If the charge on a capacitor is doubled, the energy stored becomes:

A four times

B double

C half

D unchanged

Solution: $W = \frac{1}{2} QV$ with $V = Q/C$, so W is proportional to Q^2 .

MCQ 16.7.4 The energy density of an electric field is proportional to:

A E

B $\frac{1}{E}$

C $\sqrt{E}$

D E^2

Solution: Energy density $= \frac{1}{2} \epsilon_0 \epsilon_r E^2$.

MCQ 16.7.5 The area under the V - Q graph of a capacitor gives:

A the capacitance

B the charge

C the energy stored

D the current

Solution: Each strip is $\Delta Q \times V$, a small amount of work.

Topic 16.8 Charging and Discharging a Capacitor

LQ 5 starts here — charging and discharging a capacitor with circuit diagrams and charge-time graphs, and the time constant of a capacitor discharging through a resistor

RC-Circuit

Definition

Many electric circuits contain both capacitors and resistors. A resistor-capacitor circuit is called an **RC-circuit**.

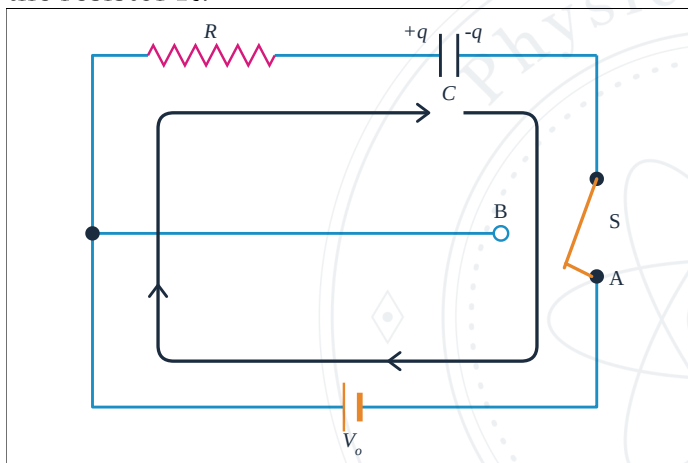
Charging a Capacitor

Definition

Charging a capacitor is the gradual building up of charge on its plates when it is connected to a battery through a resistor.

Circuit

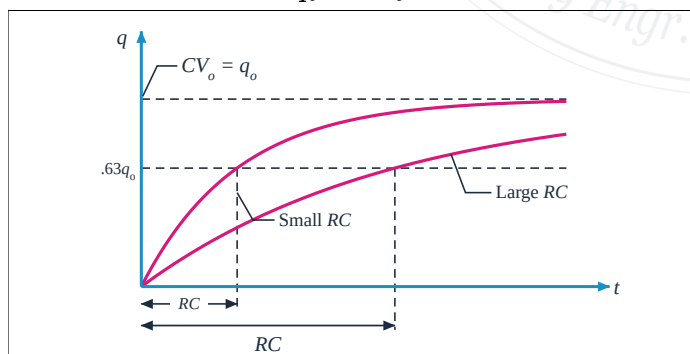
When the switch S is set at terminal A , the R - C combination is connected to a battery of voltage V_o , which charges the capacitor through the resistor R .



Growth of Charge

The capacitor is not charged immediately. The charge is zero at the start and builds up gradually to its equilibrium value.

$$q_o = CV_o$$



The voltage across the capacitor at any instant is obtained by dividing q by C

$$V = \frac{q}{C}$$

Time Constant

How fast the capacitor charges or discharges depends upon the product of the resistance R and the capacitance C . The unit of RC is that of time, so it is called the time constant: the time

required by the capacitor to deposit 0.63 times the equilibrium charge q_o .

The charge reaches its equilibrium value sooner when the time constant is small.

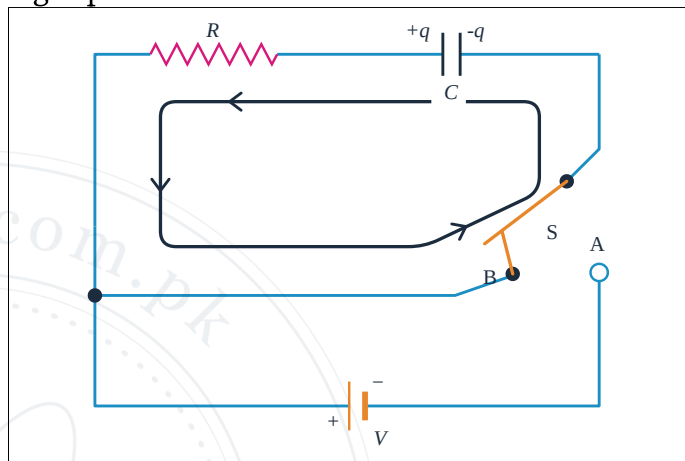
Discharging a Capacitor

Definition

Discharging a capacitor is the gradual loss of its charge when its plates are joined through a resistor, until the charges on the two plates neutralize each other.

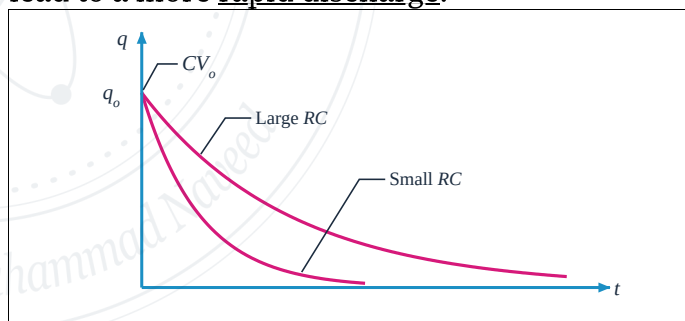
Circuit

With the switch S set at B , the charge $+q$ on the left plate flows anticlockwise through the resistance and neutralizes the charge $-q$ on the right plate.



Decay of Charge

Discharging begins at the start with the charge at CV_o and the charge decreases gradually to zero. Smaller values of the time constant RC lead to a more rapid discharge.



Book Examples

Example 16.8.1 The time constant of a series **RC-circuit** is $t = RC$. Verify that an ohm times farad is equivalent to second.

Given Data

Time constant of a series RC-circuit

$$t = RC$$

To Find

That an ohm times a farad is a second

Formula

$$V = IR$$

$$I = \frac{q}{t}$$

$$C = \frac{q}{V}$$

Solution

Putting the current in Ohm's law

$$V = \frac{q}{t} R$$

$$R = \frac{V \times t}{q}$$

Multiplying by the capacitance

$$RC = \frac{V \times t}{q} \times \frac{q}{V} = t$$

Result

The product RC has the unit of time, so an ohm times a farad is equivalent to a second.

$$1 \Omega \times 1 \text{ F} = 1 \text{ s}$$

Short Questions

SQ 16.8.1 What is an RC-circuit?

RC-circuit

A circuit containing a resistor and a capacitor, through which the capacitor is charged by a battery or discharged.

SQ 16.8.2 Why does a capacitor not charge instantly through a resistor?

Current limited by R

The resistor limits the current, so charge builds up gradually to its equilibrium value.

$$q_o = CV_o$$

SQ 16.8.3 How does the time constant affect the charging of a capacitor?

Small and large RC

With a small time constant the charge reaches its equilibrium value sooner; with a large one it grows more slowly.

MCQ 16.8.1 A 2 k Ω resistor is in series with a 50 μF capacitor. The time constant is:

- A 100 s B **0.1 s**
 C 0.01 s D 10^5 s

Solution: $RC = 2000 \times 50 \times 10^{-6} = 0.1 \text{ s}$.

MCQ 16.8.2 After one time constant of charging, the charge on a capacitor is:

- A 0.37 q_o B **0.63 q_o**
 C 0.50 q_o D q_o

Solution: The time constant is the time to deposit 0.63 of the equilibrium charge.

MCQ 16.8.3 The unit of the product RC is the:

- A **second** B ohm
 C farad D coulomb

Solution: An ohm times a farad is a second.

MCQ 16.8.4 A capacitor C charged through a resistor by a battery V_o reaches the equilibrium charge:

- A $\frac{V_o}{C}$ B $\frac{C}{V_o}$
 C $\frac{1}{2} CV_o^2$ D **CV_o**

Solution: Charging stops when the capacitor voltage equals V_o .

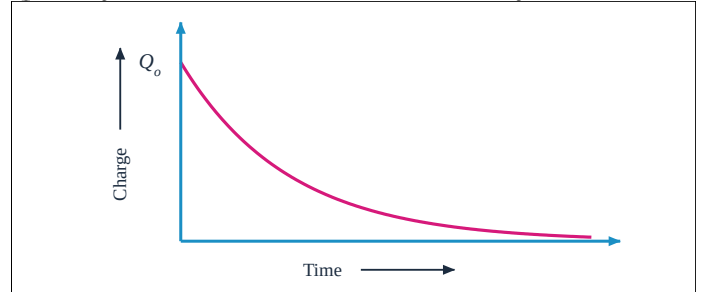
Topic 16.9 Exponential Decay of

Discharging Capacitor through Resistor

Exponential Decay

Definition

For a capacitor discharging through a resistor, the charge Q decreases exponentially over time t . An exponential process is one whose rate of change at any instant is proportional to the present value of the quantity, so it changes quickly at first and then more slowly.



Decay Formula

The general exponential decay formula for a discharging capacitor in an RC-circuit is

$$x = x_o \left[\exp \left(\frac{-t}{RC} \right) \right]$$

$$x = x_o e^{-t/RC}$$

Symbols

- x is the quantity, such as charge, voltage or current, at time t .
- x_o is the initial value of the quantity.
- R is the resistance in ohms and C the capacitance in farads.
- RC is the time constant τ , which determines how quickly the capacitor discharges.
- e is Euler's constant, the base of the natural logarithm, approximately 2.718.

The negative exponent shows that the quantity decreases exponentially over time.

Charge, Voltage and Current

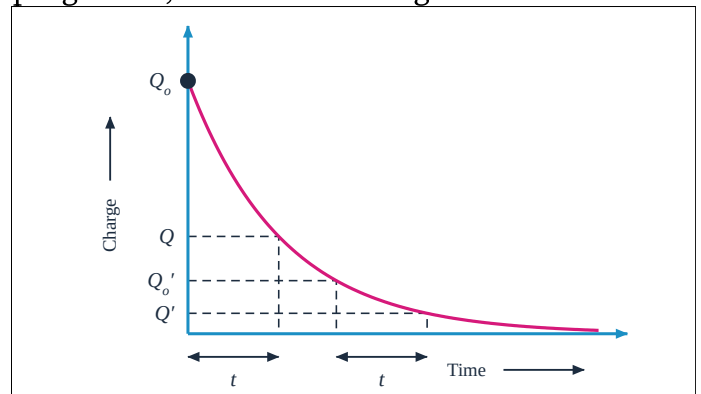
$$Q = Q_o e^{-t/RC}$$

$$V = V_o e^{-t/RC}$$

$$I = I_o e^{-t/RC}$$

Decay Graph

All three quantities start with a steep slope, a fast discharge, and gradually flatten out as time progresses, a slower discharge rate.



The initial charge Q_o drops to Q after a time interval t . At a later stage the charge Q' drops to Q' in the same interval. The decrease in the

beginning was $(Q_o - Q)$, but later it is $(Q'_o - Q')$, which is less.

Water Tank Analogy

The decay of charge is like the flow of water out of a tank through a narrow pipe. The water level and pressure are analogous to charge and voltage, and the flow rate to the current. When the level is high the water flows out fast, and as the level comes down the flow slows, because of the decreasing pressure.

For Your Information

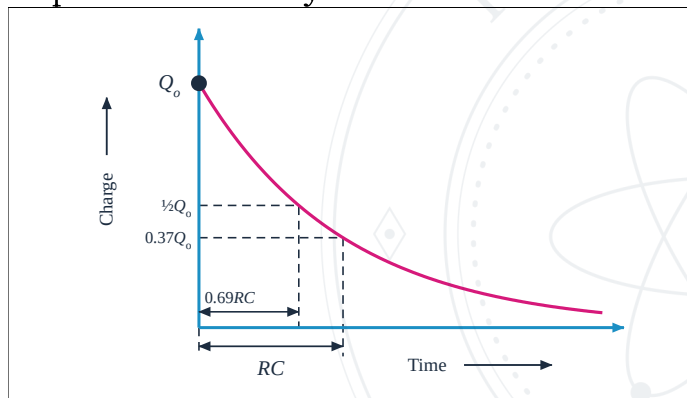
Exponential functions describe many phenomena in physics, such as radioactive decay, in which the rate of change in a process depends directly on its current value.

Time Constant

Definition

The time constant is the duration of time in which the initial charge Q_o drops to about 0.37 of Q_o . After one time constant the charge, and likewise the voltage and current, drops to 37% of its initial value.

A smaller time constant means the circuit responds faster, and a larger one that it responds more slowly.



Charge after One Time Constant

The time constant τ plays a very important role in charging and discharging a capacitor through a resistance. For the decay of charge

$$Q = Q_o e^{-t/RC}$$

Putting t equal to RC gives the charge remaining after one time constant.

$$Q = Q_o e^{-RC/RC} = Q_o e^{-1} = Q_o \times \frac{1}{e}$$

$$Q = Q_o \times \frac{1}{2.718} = 0.37Q_o$$

Half Time

Definition

The half time $t_{1/2}$ is the time in which the charge on a discharging capacitor drops to one half of its initial value.

Derivation

The time $t_{1/2}$ in which the charge Q_o drops to one half of its initial value is found by putting Q equal to one half of Q_o in the decay equation.

$$\frac{1}{2}Q_o = Q_o e^{-t_{1/2}/RC}$$

$$\frac{1}{2} = e^{-t_{1/2}/RC}$$

By definition of the natural logarithm

$$\ln\left(\frac{1}{2}\right) = \frac{-t_{1/2}}{RC}$$

$$-(RC) \left[\ln\left(\frac{1}{2}\right) \right] = t_{1/2}$$

Now

$$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = 0 - \ln(2) = -0.69$$

Putting this value

$$-RC(-0.69) = t_{1/2}$$

$$t_{1/2} = 0.69RC$$

LQ 5 ends here

Book Examples

Example 16.9.1 In an RC-circuit $R = 1 \text{ k}\Omega$, $C = 100 \text{ }\mu\text{F}$ and $V_o = 10 \text{ V}$. Determine the voltage across the capacitor after 0.2 s .

Given Data

$$R = 1 \text{ k}\Omega = 1000 \Omega$$

$$C = 100 \text{ }\mu\text{F} = 100 \times 10^{-6} \text{ F}$$

$$V_o = 10 \text{ V}$$

$$t = 0.2 \text{ s}$$

To Find

Voltage after 0.2 s $V(t) = ?$

Formula

$$\tau = RC$$

$$V = V_o e^{-t/RC}$$

Solution

The time constant is

$$\tau = RC = 1000 \Omega \times 100 \times 10^{-6} \text{ F} = 0.1 \text{ s}$$

Using the exponential decay equation for voltage

$$V = 10 \times e^{-0.2/0.1} = 10 \times e^{-2} = 10 \times \frac{1}{(2.718)^2}$$

$$= 1.35 \text{ V}$$

Result

$$V = 1.35 \text{ V}$$

Short Questions

SQ 16.9.1 What is meant by exponential decay?

Exponential decay

A decrease whose rate at any instant is proportional to the present value, so it changes quickly at first and then more slowly.

$$x = x_o e^{-t/RC}$$

SQ 16.9.2 Write the equations for the charge, voltage and current of a capacitor discharging through a resistor.

Decay equations

All three fall exponentially with the same time constant.

$$Q = Q_o e^{-t/RC}$$

$$V = V_o e^{-t/RC}$$

$$I = I_o e^{-t/RC}$$

SQ 16.9.3 How is the half time of a discharging capacitor related to its time constant?

Half time

The time in which the charge falls to one half of its initial value.

$$t_{1/2} = 0.69RC$$

SQ 16.9.4 How is the discharge of a capacitor like water flowing out of a tank?

Water tank analogy

The water level and pressure are like the charge and voltage, and the flow rate like the current. As the level falls, the pressure decreases and the flow slows down.

MCQ 16.9.1 The value of Euler's constant e is approximately:

- | | |
|----------------|--------|
| A 3.142 | B 0.37 |
| C 2.718 | D 1.6 |

Solution: It is the base of the natural logarithm.

MCQ 16.9.2 After two time constants of discharge, the charge left on a capacitor is about:

- | | |
|--------------|---------------------------------|
| A $0.37 Q_0$ | B $0.135 Q_0$ |
| C $0.27 Q_0$ | D $0.50 Q_0$ |

Solution: $Q = Q_0 e^{-2} = Q_0 / 7.39 = 0.135 Q_0$.

MCQ 16.9.3 A circuit has $RC = 10$ ms. The charge on its capacitor falls to half in:

- | | |
|-----------------|----------|
| A 10 ms | B 3.7 ms |
| C 6.9 ms | D 5 ms |

Solution: $t_{1/2} = 0.69 RC = 0.69 \times 10$ ms.

MCQ 16.9.4 In one half time, the charge of a discharging capacitor drops to:

- | | |
|-----------------------------------|----------------------------|
| A 50% of its initial value | B 37% of its initial value |
| C 63% of its initial value | D zero |

Solution: By definition of $t_{1/2}$.

MCQ 16.9.5 A capacitor at 10 V discharges with a time constant of 0.1 s. After 0.1 s its voltage is about:

- | | |
|---------|----------------|
| A 6.3 V | B 5 V |
| C zero | D 3.7 V |

Solution: $V = 10 e^{-1} = 10 \times 0.37 = 3.7$ V.

MCQ 16.9.6 A smaller time constant makes a discharging circuit respond:

- | | |
|--------------------|-----------------|
| A more slowly | B faster |
| C at the same rate | D not at all |

Solution: The charge falls to $0.37 Q_0$ in a shorter time.

Topic 16.10 Use of Capacitors

LQ 6 starts here — some uses of capacitors in household appliances

Household Appliances

Uses

Capacitors are used in many household appliances, such as electric fans, water drawing motors, washing machines, refrigerators, flashguns, computer keyboards and touchscreens.

Electric Fans and Electric Motors

A capacitor is connected to the starting of a fan motor. When the power is turned on it starts charging, and during starting it provides additional power, helping the motor overcome static friction and the initial resistance due to inertia, so the fan starts and spins quickly. Capacitors play the same role in the motors of washing machines, blenders, choppers, grinders and water drawing motors.

Refrigerators and Air-Conditioners

Their compressors use single phase motors, which can struggle to start on their own. A capacitor of high capacitance stores a big amount of electrical energy and releases it to the motor winding, a "jump start" that gets the motor rotating.

Start and Run Capacitors

Once the motor reaches a certain speed, a relay disconnects the capacitor, commonly called the start capacitor. Most refrigerators and air-conditioners also use a run capacitor, which stays connected to improve energy efficiency; some older models use only start capacitors.

Time Delay Circuit

Capacitors with resistors form a time delay circuit, used in refrigerators, air conditioners and other devices to delay triggering the power supply. This protects the devices from power fluctuation.

Flashgun

A capacitor stores electrical energy from the battery of the camera and releases it to power the flash tube, which contains xenon gas. When the shutter button is pressed, the rapid discharge of the capacitor provides the high current that ionizes the xenon, which emits a burst of bright light.

Interesting Information

A time-lapse photograph of a golfer teeing off can be taken with an electronic flash attachment on the camera; the energy for each flash comes from the electrical energy stored in a capacitor.

Other Uses

Filters as in Rectifier Circuits

Capacitors are widely used in rectifier circuits for smoothing pulsating DC into smooth DC.

Medical Uses

- Capacitors store the energy that delivers a life-saving shock to the heart, restoring a normal rhythm in cardiac arrest.
- In imaging machines (CT, MRI, X-ray and ultrasound) they filter out noise and improve image clarity.
- They filter out unwanted signals in EEG and ECG machines.

LQ 6 ends here

Short Questions

SQ 16.10.1 Why is a capacitor connected to a fan motor?

Starting the motor

On starting, it provides additional power that helps the motor overcome static friction and the initial resistance due to inertia.

SQ 16.10.2 What is the difference between a start capacitor and a run capacitor?

Start and run capacitors

A start capacitor gives the compressor motor a jump start and is disconnected by a relay at speed; a run capacitor stays in the circuit to improve energy efficiency.

SQ 16.10.3 Why is a time delay circuit used in refrigerators and air conditioners?

Protection

A capacitor with a resistor delays triggering the power supply, which protects the device from power fluctuation.

MCQ 16.10.1 The flash tube of a flashgun contains:

- | | |
|--------------------|----------------|
| A neon gas | B argon gas |
| C xenon gas | D hydrogen gas |

Solution: The capacitor's rapid discharge ionizes the xenon.

MCQ 16.10.2 In rectifier circuits, capacitors are used to:

- | | |
|-----------------------|------------------------------|
| A step up the voltage | B convert DC into AC |
| C amplify the current | D smooth pulsating DC |

Solution: They act as filters.

MCQ 16.10.3 A time delay circuit is formed by a capacitor together with a:

- | | |
|-------------------|---------------|
| A resistor | B transformer |
| C fuse | D relay |

Solution: The delay depends on the time constant RC.

MCQ 16.10.4 Once a compressor motor reaches a certain speed, its start capacitor is disconnected by a:

- | | |
|----------------|------------|
| A fuse | B diode |
| C relay | D resistor |

Solution: The start capacitor is no longer needed, while a run capacitor stays connected.

Chapter Exercise

SQ 16.1 If the absolute potential at a point is zero, what can you say about the electric intensity at that point?

The electric intensity there need not be zero.

Field and potential gradient

The intensity depends on how fast the potential changes with distance, not on the value of the potential itself.

$$E = -\frac{\Delta V}{\Delta d}$$

Midway between equal and opposite charges the potential is zero, but the field is not: it points towards the negative charge.

SQ 16.2 Is electron volt, a unit of potential difference or energy? Explain briefly.

Electron volt is a unit of energy.

Electron volt

It is the energy acquired or lost by an electron accelerated through a potential difference of 1 volt, the product of a charge and a potential difference.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

SQ 16.3 Define capacitance and write its units.

Capacitance

The amount of charge on one plate of a capacitor necessary to raise its potential by one volt with respect to the other.

$$C = \frac{Q}{V}$$

Unit

The farad (F), one coulomb per volt.

$$1 \text{ F} = 1 \text{ C V}^{-1}$$

SQ 16.4 Define dielectric constant. Give its mathematical form.

Dielectric constant

The ratio of the capacitance of a parallel plate capacitor with an insulating substance between the plates to its capacitance with vacuum (or air) between them.

$$\epsilon_r = \frac{C_{\text{med}}}{C_{\text{vac}}}$$

It has no unit.

SQ 16.5 How does the capacitance of a capacitor change if the separation between its plates is increased?

The capacitance decreases.

Inverse dependence on d

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$A\epsilon_0\epsilon_r = \text{constant}$$

$$C \propto \frac{1}{d}$$

Doubling the separation halves the capacitance.

SQ 16.6 When will the energy stored in three capacitors be greater? (i) They are connected in series. (ii) They are connected in parallel.

The energy is greater when they are connected in parallel.

Equivalent capacitance

In parallel the equivalent capacitance is the sum of all three, while in series it is less than the smallest. For the same voltage the energy grows with the capacitance.

$$W = \frac{1}{2} C_e V^2$$

SQ 16.7 What is time constant? Write its formula.

Time constant

The time required by a capacitor to deposit 0.63 times its equilibrium charge on charging, or in which the charge drops to 0.37 of its initial value on discharging.

$$\tau = RC$$

CRQ 16.1 How is electric potential different from electric potential energy? Give an example to explain the difference.

Potential and potential energy

Electric potential is the work done per unit positive charge in bringing it from infinity to a point. It belongs to the point in the field, does not depend on the test charge, and is measured in volts.

$$V = \frac{W}{q_0}$$

Electric potential energy is the energy of a particular charge placed at that point. It depends on the size of the charge and is measured in joules.

$$U = qV$$

Example

At a point where the potential is 100 V, a charge of 1 μC has 10^{-4} J of potential energy and a charge of 2 μC has 2×10^{-4} J. The potential is the same for both; the energy is not.

CRQ 16.2 How does the concept of electric potential help to understand the motion of charges in an electric field?

Direction of motion

A free positive charge always accelerates from a point of higher potential to lower potential, as a body falls from a higher to a lower level. A negative charge moves from lower to higher potential.

Energy of the motion

The potential energy lost by the charge appears as its kinetic energy, so the potential difference tells how fast it will be moving.

$$\frac{1}{2}mv^2 = q_0\Delta V$$

CRQ 16.3 The electric field is negative of gradient of potential. What does the negative sign indicate physically? Illustrate your answer with an example.

Direction of decreasing potential

The negative sign shows that the electric field points in the direction in which the potential decreases, from higher to lower potential.

$$E = -\frac{\Delta V}{\Delta d}$$

Example

Between two charged parallel plates the field points from the positive plate, at the higher potential, to the negative plate, at the lower. Around a positive point charge the potential falls as the distance grows, and the field points radially outward.

CRQ 16.4 When a capacitor is charging, no actual charge flows across the gap between the plates. But the current is still said to be flowing. Explain.

Current in the connecting wires

The battery draws electrons from the plate joined to its positive terminal and pushes an equal number onto the other plate. Charge therefore flows through the wires on both sides of the capacitor, so a current flows in the circuit, though none crosses the dielectric.

End of the current

The charge builds up on the plates until the potential difference across the capacitor equals the battery voltage; then the current stops.

CRQ 16.5 Two capacitors can be connected in parallel or in series. Explain how does total energy stored differ in two configurations.

Parallel combination

The equivalent capacitance is the sum of the two, larger than either, so with the same battery more energy is stored.

$$C_e = C_1 + C_2$$

$$W = \frac{1}{2} C_e V^2$$

Series combination

The equivalent capacitance is less than the smaller of the two, so with the same battery less energy is stored.

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

CRQ 16.6 A capacitor stores energy. Is this energy stored on the plates, in the dielectric or in the electric field? Explain how does a capacitor store energy.

The energy is stored in the electric field between the plates.

Work in charging

In charging, the battery moves charge from one plate to the other against the field, and this work is stored as the energy of the field.

$$W = \frac{1}{2} QV$$

Energy density

The energy per unit volume depends only on the field strength between the plates.

$$\text{Energy density} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

CRQ 16.7 Why does the capacitance increase on inserting a dielectric between the plates of a capacitor? Explain.

Polarization of the dielectric

The field between the plates polarizes the atoms of the dielectric, and charges appear on its surfaces opposite to those on the adjacent plates. This effectively decreases the charge on the plates, so the field and the potential difference between them decrease.

Result

The charge Q is unchanged while V decreases, so the capacitance increases by the factor ϵ_r

$$C = \frac{Q}{V}$$

$$C_{med} = \epsilon_r C_{vac}$$

MCQ 16.1 What is the work done on an electron by a potential difference of 100 volts?

- | | | | |
|---|---------------------------|---|--------------------------|
| A | 1.6×10^{-19} eV | B | 1.6×10^{-17} eV |
| C | 6.25×10^{-17} eV | D | 100 eV |

Solution: $W = qV = e \times 100 \text{ V} = 100 \text{ eV}$, since 1 eV is the energy of an electron through 1 V.

MCQ 16.2 A parallel plate capacitor of capacitance 50 μF is connected to a battery of 12 volts, the charge stored on each plate is:

- | | | | |
|---|----------------------|---|-------------------|
| A | 60 μC | B | 4.2 μC |
| C | 6×10^{-4} C | D | 4×10^4 C |

Solution: $Q = CV = 50 \times 10^{-6} \text{ F} \times 12 \text{ V} = 6 \times 10^{-4} \text{ C}$.

MCQ 16.3 If a dielectric is inserted between the plates of a charged capacitor, the potential difference across the plates:

- | | | | |
|---|--------------|---|--------------|
| A | increases | B | decreases |
| C | remains same | D | becomes zero |

Solution: Polarization of the dielectric reduces the field between the plates, so V falls while Q stays the same.

MCQ 16.4 The potential energy of a system of two charges 8 μC and $-8 \mu\text{C}$ separated by a distance 2 m is:

- | | | | |
|---|---|---|--|
| A | $\frac{8 \times 10^{-12}}{\pi \epsilon_0} \text{ J}$ | B | $\frac{+16 \times 10^{-12}}{\pi \epsilon_0} \text{ J}$ |
| C | $\frac{-8 \times 10^{-12}}{\pi \epsilon_0} \text{ J}$ | D | $\frac{-16 \times 10^{-12}}{\pi \epsilon_0} \text{ J}$ |

Solution: $U = (1/4\pi\epsilon_0)(8 \times 10^{-6})(-8 \times 10^{-6})/2 = -64 \times 10^{-12}/(8\pi\epsilon_0) = -8 \times 10^{-12}/(\pi\epsilon_0) \text{ J}$.

MCQ 16.5 The electric potential at point A is 50 V and at B is 80 V. If the electric field between the charges is uniform, what can be said about the motion of positive charge from A to B?

- | | | | |
|---|---|---|------------------------------------|
| A | No work is done | B | Work is done by the electric field |
| C | Work is done against the electric field | D | None of the above |

Solution: A positive charge moving from lower to higher potential gains potential energy, so work is done on it against the field.

MCQ 16.6 The rate of charging or discharging the capacitor depends upon the product of resistance and:

- | | | | |
|---|--------|---|----------------------|
| A | charge | B | potential difference |
|---|--------|---|----------------------|

- | | | | |
|---|---------|---|-------------|
| C | current | D | capacitance |
|---|---------|---|-------------|

Solution: The time constant is RC .

MCQ 16.7 In a charged capacitor, the energy resides in:

- | | | | |
|---|-----------------------------------|---|---------------------------------------|
| A | the positive plate | B | the negative plate |
| C | both positive and negative plates | D | the electric field between the plates |

Solution: The energy density is $\frac{1}{2} \epsilon_0 \epsilon_r E^2$, the energy of the field between the plates.

MCQ 16.8 A capacitor of capacitance 1 μF is subjected to 4000 volts potential difference, the energy stored in it is:

- | | | | |
|---|------|---|------|
| A | 8 J | B | 16 J |
| C | 32 J | D | 64 J |

Solution: $W = \frac{1}{2} CV^2 = \frac{1}{2} \times 10^{-6} \text{ F} \times (4000 \text{ V})^2 = 8 \text{ J}$.

MCQ 16.9 The electric potential energy of a point charge q at some point in an electric field is equal to:

- | | | | |
|---|--|---|--|
| A | $\frac{1}{4\pi\epsilon_0} \frac{q}{r}$ | B | $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ |
| C | qV | D | V^2 |

Solution: The work done in bringing q to a point of potential V is qV , which is stored as its potential energy.

MCQ 16.10 The time constant of a capacitor discharging through a resistor is the time in seconds in which the initial charge Q_0 drops to:

- | | | | |
|---|------------|---|------------|
| A | $0.25 Q_0$ | B | $0.37 Q_0$ |
| C | $0.50 Q_0$ | D | $0.69 Q_0$ |

Solution: At $t = RC$ the charge is $Q_0 e^{-1} = Q_0/2.718 = 0.37 Q_0$.

Numerical 16.1 A particle carrying a charge of 0.1 μC starts from rest in a uniform electric field of intensity 60 N C^{-1} . Find the kinetic energy acquired by the particle, when it has moved 2 m. (Ans: $1.2 \times 10^{-5} \text{ J}$)

Given Data

$$q = 0.1 \mu\text{C} = 1 \times 10^{-7} \text{ C}$$

$$E = 60 \text{ N C}^{-1}$$

$$d = 2 \text{ m}$$

To Find

Kinetic energy acquired $K.E. = ?$

Formula

The work done by the field appears as kinetic energy.

$$F = qE$$

$$K.E. = W = Fd = qEd$$

Solution

Putting the values in the formula

$$K.E. = 1 \times 10^{-7} \text{ C} \times 60 \text{ N C}^{-1} \times 2 \text{ m} \\ = 1.2 \times 10^{-5} \text{ J}$$

Result $K.E. = 1.2 \times 10^{-5} \text{ J}$

Numerical 16.2 What should be the potential difference through which an electron falls to acquire a speed of $2 \times 10^6 \text{ m s}^{-1}$? (Ans: 11.4 V)

Given Data

$$v = 2 \times 10^6 \text{ m s}^{-1}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

To Find

Potential difference $\Delta V = ?$

Formula $\frac{1}{2}mv^2 = e\Delta V$

$$\Delta V = \frac{mv^2}{2e}$$

Solution

Putting the values in the formula

$$\Delta V = \frac{9.1 \times 10^{-31} \times (2 \times 10^6)^2}{2 \times 1.6 \times 10^{-19}}$$

$$= 11.4 \text{ V}$$

Result $\Delta V = 11.4 \text{ V}$

Numerical 16.3 The potential difference between points A and B is $V_A - V_B = 84 \text{ V}$, while the potential difference between points C and B is $V_C - V_B = 24 \text{ V}$. What should be the work done on moving a $40 \mu\text{C}$ charge from C to A? (Ans: $2.4 \times 10^{-3} \text{ J}$)

Given Data

$$V_A - V_B = 84 \text{ V}$$

$$V_C - V_B = 24 \text{ V}$$

$$q = 40 \mu\text{C} = 40 \times 10^{-6} \text{ C}$$

To Find

Work done from C to A $W = ?$

Formula

$$V_A - V_C = (V_A - V_B) - (V_C - V_B)$$

$$W = q(V_A - V_C)$$

Solution

The potential difference between A and C is

$$V_A - V_C = 84 \text{ V} - 24 \text{ V} = 60 \text{ V}$$

The work done is

$$W = 40 \times 10^{-6} \text{ C} \times 60 \text{ V}$$

$$= 2.4 \times 10^{-3} \text{ J}$$

Result $W = 2.4 \times 10^{-3} \text{ J}$

Numerical 16.4 A parallel plate capacitor has a capacitance of $6 \mu\text{F}$ when filled with a dielectric. The area of each plate is 1.2 m^2 and the separation between the plates is $9 \times 10^{-6} \text{ m}$. What is the dielectric constant? (Ans: 5.1)

Given Data

$$C_{\text{med}} = 6 \mu\text{F} = 6 \times 10^{-6} \text{ F}$$

$$A = 1.2 \text{ m}^2$$

$$d = 9 \times 10^{-6} \text{ m}$$

$$\epsilon_o = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

To Find

Dielectric constant $\epsilon_r = ?$

Formula

$$C_{\text{med}} = \frac{A\epsilon_o\epsilon_r}{d}$$

$$\epsilon_r = \frac{C_{\text{med}}d}{A\epsilon_o}$$

Solution

Putting the values in the formula

$$\epsilon_r = \frac{6 \times 10^{-6} \times 9 \times 10^{-6}}{1.2 \times 8.85 \times 10^{-12}}$$

$$= 5.1$$

Result $\epsilon_r = 5.1$

Numerical 16.5 A $15 \mu\text{F}$ capacitor is charged till the potential difference across its plates becomes 300 V . Find the energy stored in it. (Ans: 0.675 J)

Given Data

$$C = 15 \mu\text{F} = 15 \times 10^{-6} \text{ F}$$

$$V = 300 \text{ V}$$

To Find Energy stored $W = ?$

Formula $W = \frac{1}{2}CV^2$

Solution

Putting the values in the formula

$$W = \frac{1}{2} \times 15 \times 10^{-6} \text{ F} \times (300 \text{ V})^2$$

$$= 0.675 \text{ J}$$

Result $W = 0.675 \text{ J}$

Numerical 16.6 Two capacitors of $2.0 \mu\text{F}$ and $8.0 \mu\text{F}$ capacitances are connected in series and a potential difference of 300 volts is applied. Find the charge and potential difference for each capacitor. (Ans: $Q_1 = Q_2 = 4.8 \times 10^{-4} \text{ C}$, $V_1 = 240 \text{ V}$, $V_2 = 60 \text{ V}$)

Given Data

$$C_1 = 2.0 \mu\text{F} = 2 \times 10^{-6} \text{ F}$$

$$C_2 = 8.0 \mu\text{F} = 8 \times 10^{-6} \text{ F}$$

$$V = 300 \text{ V}$$

To Find

Charges Q_1 and Q_2 , and potential differences V_1 and V_2

Formula

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q = C_e V$$

$$V_1 = \frac{Q}{C_1}$$

Solution

The equivalent capacitance is

$$\frac{1}{C_e} = \frac{1}{2\mu\text{F}} + \frac{1}{8\mu\text{F}}$$

$$= \frac{5}{8\mu\text{F}}$$

$$C_e = 1.6\mu\text{F} = 1.6 \times 10^{-6}\text{ F}$$

In series each capacitor carries the same charge.

$$Q_1 = Q_2 = C_e V = 1.6 \times 10^{-6}\text{ F} \times 300\text{ V}$$

$$= 4.8 \times 10^{-4}\text{ C}$$

$$V_1 = \frac{4.8 \times 10^{-4}\text{ C}}{2 \times 10^{-6}\text{ F}}$$

$$= 240\text{ V}$$

$$V_2 = \frac{4.8 \times 10^{-4}\text{ C}}{8 \times 10^{-6}\text{ F}}$$

$$= 60\text{ V}$$

Result

$$Q_1 = Q_2 = 4.8 \times 10^{-4}\text{ C}$$

$$V_1 = 240\text{ V}$$

$$V_2 = 60\text{ V}$$

Numerical 16.7 A capacitor has a capacitance of $3\mu\text{F}$. It is charged by a battery. When the potential difference between the plates is 240 volts, how many electrons have been removed from one plate and placed on the other plate? (Ans: 4.5×10^{15} electrons)

Given Data

$$C = 3\mu\text{F} = 3 \times 10^{-6}\text{ F}$$

$$V = 240\text{ V}$$

$$e = 1.6 \times 10^{-19}\text{ C}$$

To Find

Number of electrons $n = ?$

Formula $Q = CV$

$$Q = ne$$

$$n = \frac{CV}{e}$$

Solution

The charge on each plate is

$$Q = 3 \times 10^{-6}\text{ F} \times 240\text{ V}$$

$$= 7.2 \times 10^{-4}\text{ C}$$

The number of electrons is

$$n = \frac{7.2 \times 10^{-4}\text{ C}}{1.6 \times 10^{-19}\text{ C}}$$

$$= 4.5 \times 10^{15}$$

Result

$$n = 4.5 \times 10^{15}\text{ electrons}$$

Numerical 16.8 Two positive point charges are held at two points 1.2 m apart. They are then moved towards each other so that their electric potential energy becomes double of its previous value. How much are they apart at their new positions? (Ans: 0.6 m)

Given Data

$$r = 1.2\text{ m}$$

$$U' = 2U$$

To Find New separation $r' = ?$

Formula

$$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

$$U' = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r'}$$

Dividing the two

$$\frac{U'}{U} = \frac{r}{r'}$$

$$r' = r \frac{U}{U'}$$

Solution

Putting the values

$$r' = 1.2\text{ m} \times \frac{U}{2U} = 0.6\text{ m}$$

Result $r' = 0.6\text{ m}$

Numerical 16.9 A $100\mu\text{F}$ capacitor is initially charged to 8 V. It is allowed to discharge through a 500Ω resistor. Find the voltage across the capacitor after 0.5 s. (Ans: $3.6 \times 10^{-4}\text{ V}$)

Given Data

$$C = 100\mu\text{F} = 100 \times 10^{-6}\text{ F}$$

$$V_o = 8\text{ V}$$

$$R = 500\Omega$$

$$t = 0.5\text{ s}$$

To Find

Voltage after 0.5 s $V = ?$

Formula $\tau = RC$

$$V = V_o e^{-t/RC}$$

Solution

The time constant is

$$\tau = RC = 500\Omega \times 100 \times 10^{-6}\text{ F} = 0.05\text{ s}$$

The voltage after 0.5 s is

$$V = 8 \times e^{-0.5/0.05} = 8 \times e^{-10} = 8 \times 4.54 \times 10^{-5}$$

$$= 3.6 \times 10^{-4}\text{ V}$$

Result $V = 3.6 \times 10^{-4}\text{ V}$

Self Test

Seventeen questions across the whole chapter. Work through them without the notes, then check the answers keyed at the foot of the page.

Q 1 Electrostatics deals with the phenomena of electric charges that are:

A	moving with constant velocity	B	accelerating
C	stationary	D	flowing in a conductor

Q 2 The work done in moving a charge of 2 C through a potential difference of 10 V is:

A	5 J	B	0.2 J
C	12 J	D	20 J

Q 3 In practice, the absolute value of the electric potential at a point is measured with respect to:

A	the nearest charge	B	the Earth
C	the positive terminal of a battery	D	the centre of the field

Q 4 The electric potential at 0.5 m from a charge of 1 μC is:

A	$1.8 \times 10^4 \text{ V}$	B	$4.5 \times 10^3 \text{ V}$
C	$3.6 \times 10^4 \text{ V}$	D	$9 \times 10^3 \text{ V}$

Q 5 Two point charges have potential energy U. If one of the charges is doubled at the same separation, the potential energy becomes:

A	U/2	B	4U
C	2U	D	U

Q 6 A body of mass m and charge q_0 starts from rest and falls through a potential difference ΔV . Its speed becomes:

A	$q_0 \Delta V / m$	B	$\sqrt{2q_0 \Delta V / m}$
C	$2q_0 \Delta V / m$	D	$\sqrt{q_0 \Delta V / 2m}$

Q 7 Treating the Earth as an isolated sphere of radius $6.4 \times 10^6 \text{ m}$, its capacitance is about:

A	7.1 F	B	$7.1 \times 10^{-12} \text{ F}$
C	64 F	D	$7.1 \times 10^{-4} \text{ F}$

Q 8 The ratio $C_{\text{med}}/C_{\text{vac}}$ of a parallel plate capacitor is called the:

A	dielectric constant	B	permittivity of free space
C	surface charge density	D	time constant

Q 9 The charges that appear on the faces of a dielectric slab next to the plates are:

A	opposite in sign to the adjacent plate	B	of the same sign as the adjacent plate
C	zero	D	positive on both faces

Q 10 Capacitors of 3 μF and 6 μF are connected in parallel across 10 V. The total charge stored is:

A	20 μC	B	90 μC
C	30 μC	D	60 μC

Q 11 n identical capacitors, each of capacitance C, are connected in series. The equivalent capacitance is:

A	nC	B	C
C	C/n	D	n/C

Q 12 The energy stored in a capacitor C holding a charge Q is:

A	$\frac{Q^2 C}{2}$	B	$\frac{2Q^2}{C}$
C	$\frac{QC}{2}$	D	$\frac{Q^2}{2C}$

Q 13 If the electric field between the plates of a capacitor is doubled, the energy density becomes:

A	double	B	half
C	four times	D	one quarter

Q 14 At the instant the switch of an RC charging circuit is closed, the charge on the capacitor is:

A	q_0	B	zero
C	$0.63 q_0$	D	$0.37 q_0$

Q 15 The voltage of a discharging capacitor falls as:

A	$V_0 e^{t/RC}$	B	$V_0 (1 - e^{-t/RC})$
C	$V_0 t / RC$	D	$V_0 e^{-t/RC}$

Q 16 The natural logarithm of 2 is approximately:

A	0.69	B	0.37
C	0.63	D	2.718

Q 17 The capacitor that stays connected in a refrigerator to improve energy efficiency is the:

A	start capacitor	B	filter capacitor
C	run capacitor	D	flash capacitor

Answers 1 (c) 2 (d) 3 (b) 4 (a) 5 (c) 6 (b) 7 (d) 8 (a) 9 (a) 10 (b) 11 (c) 12 (d) 13 (c) 14 (b) 15 (d) 16 (a) 17 (c)