TOPIC 1 Electric Potential Difference

SQ 16.1.1 Define electric potential difference.

Definition

The electric potential difference is the work done per unit charge in moving it between two points against the electric field.

Formula

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0} = \frac{\Delta U}{q_0}$$

SQ 16.1.2 Write the SI unit of potential difference.

SI Unit

Its SI unit is the volt, where

$$1\,\mathrm{V} = \frac{1\,\mathrm{J}}{1\,\mathrm{C}}.$$

It can be converted into other units such as kilovolts or millivolts.

SQ 16.1.3 Write the relation between potential difference and electric field.

Formula

$$\Delta V = -\mathbf{E} \cdot \Delta \mathbf{d}$$

SQ 16.1.4 Define potential gradient and write the field in terms of it.

Definition

The electric field represents the rate of change of potential with distance, which is called the gradient of potential.

Formula

$$E = -\frac{\Delta V}{\Delta d}$$

SQ 16.1.5 What does the negative sign in E = −ΔV/Δd indicate?

Answer

The negative sign indicates that the field points from higher to lower potential.

SQ 16.1.6 Write the relation for a uniform electric field.

Formula

$$E = \frac{V}{d}$$

This shows that the potential difference between two points is directly proportional to the electric field strength and the distance between them.

SQ 16.1.7 What does a stronger electric field cause?

Answer

A stronger electric field causes a more rapid decrease in potential, providing a clear physical link between field intensity and voltage variation in space.

SQ 16.1.8 Why is the Earth assumed to be at zero potential?

Answer

Although points at infinity are assumed to be at zero potential, practically the Earth is assumed to be at zero potential. Therefore the absolute value of electric potential at a point is measured with respect to the Earth.

SQ 16.1.9 A 6 V battery is connected to plates 0.3 cm apart. Find the field intensity between them.

Solution

Using

$$E = \frac{\Delta V}{\Delta d}$$

Calculation

$$E = \frac{6}{0.003}$$

Result

$$E = 2000\,\mathrm{V\,m^{-1}}$$

TOPIC 2 Electric Potential

SQ 16.2.1 Define electric potential at a point.

Definition

The electric potential at any point in an electric field is equal to the work done in bringing a unit positive charge from infinity to that point, keeping it in electrostatic equilibrium.

Formula

$$V = \frac{W}{q_0}$$

SQ 16.2.2 Why are potential and potential difference scalar quantities?

Reason

Both potential and potential difference are scalar quantities because both the work W and the charge q_0 are scalars.

SQ 16.2.3 Is the potential at a point really a potential difference?

Answer

Yes. The potential at a point is still the potential difference between the potential at that point and the potential at infinity.

SQ 16.2.4 Why can ΔV = −EΔr not be used directly for a point charge?

Reason

The relation requires the electric intensity E to remain constant. In the field of a point charge, E varies inversely as the square of the distance, so it no longer remains constant.

SQ 16.2.5 How is the difficulty of a varying field overcome in deriving potential?

Method

Two points A and B are taken infinitesimally close to each other, so that E remains almost constant between them.

SQ 16.2.6 **Which approximation is used for the mid-point distance in the derivation?**

Approximation

As the points A and B are very close, as a first approximation the arithmetic mean is taken to be equal to the geometric mean, which gives

$r^2 = r_A r_B$

SQ 16.2.7 **Write the potential difference between two nearby points in a radial field.**

Formula

$$V_A - V_B = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

SQ 16.2.8 **Derive the absolute electric potential due to a point charge.**

Derivation

To calculate the absolute potential at A, the point B is assumed to be at infinity so that

$V_B = 0$

and

$$\frac{1}{r_B} = 0$$

Result

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

SQ 16.2.9 **When is the electric potential negative?**

Answer

The electric potential will be negative if the charge q is negative.

SQ 16.2.10 **How is the total potential found when there are several charges?**

Answer

Each charge contributes to the total electric potential. At any point the total electric potential is the algebraic sum of the individual potentials created by all the charges.

SQ 16.2.11 **Find the electric potential 1.2 m from a charge of 5.0 × 10⁻⁸ C.**

Solution

Using

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

Calculation

$$V = \frac{9 \times 10^9 \times 5.0 \times 10^{-8}}{1.2}$$

Result

$V = 375\,\mathrm{V}$

SQ 16.2.12 **For a pair of unequal opposite charges, where are the zero potential points?**

Answer

The potential is zero where the potential due to the positive charge is exactly equal in magnitude to that due to the negative charge.

Location

Both points lie closer to the smaller charge, one between the two charges and the other outside the negative charge.

TOPIC 3 Electric Potential Energy

SQ 16.3.1 **How does a positive charge move freely in an electric field?**

Answer

If a positive charge is allowed to move freely in an electric field, it will always accelerate from a point of higher potential to a point of lower potential.

SQ 16.3.2 **What happens to the potential energy of a freely moving charge?**

Answer

The potential energy of the charge decreases by

$\Delta U = q_0 \Delta V$

This decrease in potential energy appears as its kinetic energy.

SQ 16.3.3 **Write the velocity gained by a charge falling through a potential difference.**

Formula

$$q_0 \Delta V = \frac{1}{2} m v^2$$

SQ 16.3.4 **How does a negative charge move in an electric field?**

Answer

A negative charge allowed to move freely in an electric field moves from a point of lower potential to a point of higher potential.

SQ 16.3.5 **Define the electron volt and give its value.**

Definition

The electron volt is the energy acquired or lost by an electron which has been accelerated through a potential difference of 1 volt.

Value

$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

SQ 16.3.6 **Derive the electric potential energy of two point charges.**

Derivation

A charge Q at point A creates a potential V at point B. If a charge q is brought from infinity to B, the work done is

$W = qV$

Result

$$U = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$$

SQ 16.3.7 **What is stored as electric potential energy in a two-charge system?**

Answer

By definition, the work done in placing the two charges at points A and B is stored as electric potential energy in the system.

SQ 16.3.8 **Two charges of 8 μC and 5 μC are 4.0 cm apart. Find their potential energy.**

Solution

Using

$$U = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$$

Calculation

$$U = \frac{9 \times 10^9 \times 8 \times 10^{-6} \times 5 \times 10^{-6}}{4 \times 10^{-2}}$$

Result

$U = 9\,\text{J}$

TOPIC 4 Capacitors

SQ 16.4.1 Define a capacitor.

Definition

A capacitor is a device that can store electrical energy in the form of an electric field.

SQ 16.4.2 Name the two types of capacitors discussed in the book.

Types

Isolated spherical conductors and parallel plate capacitors.

SQ 16.4.3 Define an isolated spherical conductor.

Definition

A spherical conductor placed in space such that it is far from other conductors or influence is known as an isolated spherical conductor.

SQ 16.4.4 Define capacitance of an isolated spherical conductor.

Definition

The capacitance of an isolated hollow spherical conductor is the electric charge stored on the sphere per unit electric potential at its surface.

Formula

$$C = \frac{Q}{V}$$

SQ 16.4.5 Define one farad for a spherical conductor.

Definition

For a spherical conductor, 1 farad is the capacitance of a sphere that stores 1 coulomb of charge when its electric potential is 1 volt, assuming it is isolated in free space.

SQ 16.4.6 Derive the capacitance of an isolated spherical conductor in vacuum.

Derivation

Since

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

substituting in

$$C = \frac{Q}{V}$$

gives

Result

$$C_{vac} = 4\pi\varepsilon_0 R$$

SQ 16.4.7 Write the capacitance of a spherical conductor in a medium.

Formula

$$C_{med} = 4\pi\varepsilon_0\varepsilon_r R$$

Here ε_r is the dielectric constant of the medium.

SQ 16.4.8 On what does the capacitance of a spherical conductor depend?

Answer

The capacitance of a spherical conductor is independent of the charge or potential. It depends only on the radius of the sphere and the medium.

SQ 16.4.9 What is the effect of increasing the radius of a spherical conductor?

Answer

The capacitance increases with an increase in the radius of the sphere. This means larger spheres store more charge at the same surface potential.

TOPIC 5 Parallel Plate Capacitors

SQ 16.5.1 Describe a parallel plate capacitor.

Description

A parallel plate capacitor consists of two conductors placed near one another, separated by vacuum, air or any other insulator known as a dielectric. The conductors are usually in the form of parallel plates.

SQ 16.5.2 What happens when a parallel plate capacitor is connected to a battery?

Answer

It establishes a potential difference of V volts between the two plates. The battery places a charge $+Q$ on the plate connected to its positive terminal and $-Q$ on the other plate.

SQ 16.5.3 Define the capacitance of a capacitor.

Definition

The capacitance of a capacitor is the amount of charge on one plate necessary to raise the potential of that plate by one volt with respect to the other.

Formula

$$C = \frac{Q}{V}$$

SQ 16.5.4 What does capacitance measure?

Answer

Capacitance depends upon the geometry of the plates and the medium between them. It is a measure of the ability of a capacitor to store charge.

SQ 16.5.5 Why must the separation between the plates be small?

Reason

The distance d is kept small so that the electric field between the plates is uniform and confined almost entirely in the region between the plates.

SQ 16.5.6 Derive the capacitance of a parallel plate capacitor in vacuum.

Derivation

Since

$$E = \frac{V}{d}$$

and by Gauss's law

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Equating and substituting in

$$C = \frac{Q}{V}$$

Result

$$C = \frac{\varepsilon_0 A}{d}$$

SQ 16.5.7 Write the capacitance of a parallel plate capacitor with a dielectric.

Formula

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

The capacitance is enhanced by the factor ε_r.

SQ 16.5.8 Describe the experiment showing the effect of a dielectric.

Experiment

A charged capacitor has its plates connected to a voltmeter. When a dielectric material is inserted between the plates, the reading drops, indicating a decrease in the potential difference.

Conclusion

Since

$$C = \frac{Q}{V}$$

and V decreases while Q remains constant, the value of C increases.

SQ 16.5.9 Define dielectric constant.

Definition

The dielectric constant is the ratio of the capacitance of a parallel plate capacitor with an insulating substance as medium between the plates, to its capacitance with vacuum or air as medium.

Formula

$$\varepsilon_r = \frac{C_{med}}{C_{vac}}$$

SQ 16.5.10 What is polarization of a dielectric?

Definition

A dielectric is an insulator with no free electrons. When such an atom is subjected to an electric field, its electrons are displaced slightly relative to the nucleus in a direction opposite to the field. This phenomenon is known as polarization, and the distorted atom is said to be polarized.

SQ 16.5.11 Why does the potential difference decrease when a dielectric is inserted?

Reason

The atoms of the dielectric get polarized, so charges appear on those surfaces of the slab in contact with the metal plates. The polarity of these charges is opposite to that of the adjacent plate.

Result

This effectively decreases the charge on the plates, so the electric intensity and the potential difference decrease.

SQ 16.5.12 How does the capacitance change if the separation between the plates is increased?

Answer

Since

$$C = \frac{\varepsilon_0 A}{d}$$

the capacitance is inversely proportional to the separation. Therefore increasing the separation decreases the capacitance.

TOPIC 6 Combined Capacitance of Capacitors

SQ 16.6.1 What is a combination of capacitors and why is it made?

Definition

When two or more capacitors are connected together in a circuit, it is known as a combination of capacitors.

Purpose

It is done to obtain a desired capacitance, and multiple connections behave as a single equivalent capacitor.

SQ 16.6.2 Name the two common types of combination of capacitors.

Types

Parallel combination and series combination.

SQ 16.6.3 Describe the parallel combination of capacitors.

Description

The plates of one side of all the capacitors are connected to the negative terminal and the plates of the other side to the positive terminal of the battery.

Property

The potential difference across each capacitor is the same, while the charge on each will be different depending upon its capacitance.

SQ 16.6.4 Derive the equivalent capacitance for a parallel combination.

Derivation

Since the total charge is

$$Q = Q_1 + Q_2 + Q_3$$

and each capacitor has the same potential difference V, therefore

$$C_e V = C_1 V + C_2 V + C_3 V.$$

Result

$$C_e = C_1 + C_2 + C_3$$

SQ 16.6.5 State the rule for capacitors connected in parallel.

Rule

When capacitors are connected in parallel, their combined capacitance is equal to the sum of their individual capacitances.

SQ 16.6.6 Describe the series combination of capacitors.

Description

The capacitors are connected one after the other in a line, with the second plate of one capacitor connected to the first plate of the next.

SQ 16.6.7 Why does each capacitor in series carry the same charge?

Reason

The battery sends charge $+Q$ to the first plate of C_1. Since the second plate is insulated it cannot draw charge from the battery, but the positive charge on the first plate induces an equal negative charge on it.

Result

Thus each capacitor receives a charge of magnitude Q on each of its plates.

SQ 16.6.8 **Write the potential relation for a series combination.**

Formula

$V = V_1 + V_2 + V_3$

The potential difference across each capacitor is different, depending upon its capacitance.

SQ 16.6.9 **Write the equivalent capacitance for a series combination.**

Formula

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

SQ 16.6.10 **An 80 pF capacitor charged to 48 V is connected in parallel to another, dropping to 30 V. Find the charge shifted.**

Solution

Initial charge

$Q_1 = 80 \times 10^{-12} \times 48 = 3840 \times 10^{-12}\,\mathrm{C}$

and the new charge on C_1 is 2400×10^{-12} C.

Result

$Q_2 = 1440 \times 10^{-12}\,\mathrm{C}$

SQ 16.6.11 **For the above problem, find the capacitance of the second capacitor.**

Solution

$$C_2 = \frac{Q_2}{V} = \frac{1440 \times 10^{-12}}{30}$$

Result

$C_2 = 48\,\mathrm{pF}$

SQ 16.6.12 **Find the equivalent capacitance of 4 μF and 8 μF connected in series.**

Solution

$$\frac{1}{C_e} = \frac{1}{4} + \frac{1}{8}$$

Result

$C_e = 2.67\,\mu\mathrm{F}$

TOPIC 7 Energy Stored in a Capacitor

SQ 16.7.1 **How is a capacitor also a device for storing energy?**

Answer

A capacitor is a device to store charge. Alternatively, it is possible to think of a capacitor as a device for storing electrical energy.

SQ 16.7.2 **Give evidence of energy stored in a charged capacitor.**

Evidence

If we connect the two plates of a charged capacitor by a thick wire, the charge flows from one plate to the other and the wire becomes hot.

Conclusion

Since heat is a form of energy, the capacitor possessed energy when it was charged.

SQ 16.7.3 **Why is the graph of V against Q a straight line?**

Reason

The charge Q on the plates increases with the increase in potential V, that is $Q \propto V$. Therefore the graph is a straight line passing through the origin.

SQ 16.7.4 **How is the energy stored found graphically?**

Method

The work required to move a small charge ΔQ through a potential difference V is

$\Delta W = (\Delta Q)V$

which is the area of a thin rectangle under the graph.

Result

The total work is the area of the triangle under the V against Q graph.

SQ 16.7.5 **Write the three expressions for energy stored in a capacitor.**

Formulas

$$W = \frac{1}{2}QV$$

$$W = \frac{1}{2}CV^2$$

$$W = \frac{Q^2}{2C}$$

SQ 16.7.6 **Where is the energy of a charged capacitor stored?**

Answer

Energy can be considered to be stored in the electric field between the plates, rather than as the potential energy of the charges on the plates.

SQ 16.7.7 **Define energy density of a capacitor.**

Definition

Energy density is the energy stored per unit volume between the plates.

Formula

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

This equation is valid for any electric field strength.

SQ 16.7.8 **Two capacitors of 4 μF and 8 μF in series are charged by 120 V. Find the total charge.**

Solution

The equivalent capacitance is $2.67\,\mu$F, so

$Q = C_e V$

Calculation

$Q = 2.67 \times 10^{-6} \times 120$

Result

$Q = 3.2 \times 10^{-4}\,\mathrm{C}$

SQ 16.7.9 **A 16 μF capacitor is charged to 100 V. Find the energy stored.**

Solution

$$W = \frac{1}{2}CV^2 = \frac{1}{2}(16 \times 10^{-6})(100)^2$$

Result

$W = 0.08\,\mathrm{J}$

SQ 16.7.10 **By what factor does the energy increase if the potential difference is doubled?**

Answer

Since the energy depends on V^2, if V is doubled the energy increases by a factor of 4.

SQ 16.7.11 **How does a defibrillator use a capacitor?**

Answer

A device called a defibrillator uses the charge stored in a capacitor to deliver a controlled electric shock that can restore normal heart rhythm.

TOPIC 8 Charging and Discharging a Capacitor

SQ 16.8.1 **What is an RC circuit?**

Definition

Many electric circuits consist of both capacitors and resistors. Such a resistor-capacitor circuit is called an RC circuit.

SQ 16.8.2 **How does a capacitor charge in an RC circuit?**

Answer

The capacitor is not charged immediately; rather the charge builds up gradually to the equilibrium value

$q_0 = CV_0$

SQ 16.8.3 **How is the voltage across a charging capacitor obtained?**

Answer

The voltage across the capacitor at any instant is obtained by dividing the charge by the capacitance, that is

$V = \frac{q}{C}$

SQ 16.8.4 **On what does the rate of charging or discharging depend?**

Answer

How fast or how slow the capacitor charges or discharges depends upon the product of the resistance and the capacitance used in the circuit.

SQ 16.8.5 **Define the time constant of an RC circuit.**

Definition

Since the unit of the product RC is that of time, this product is known as the time constant. It is the time required by the capacitor to deposit 0.63 times the equilibrium charge.

SQ 16.8.6 **What is the effect of a small time constant on charging?**

Answer

The charge reaches its equilibrium value sooner when the time constant is small.

SQ 16.8.7 **Describe the discharging of a capacitor through a resistor.**

Answer

When the switch is set so that the capacitor is connected across the resistor, the charge on the left plate flows through the resistance and neutralizes the charge on the right plate.

Graph

Discharging begins at

$t = 0$

when

$q = CV_0$

and decreases gradually to zero.

SQ 16.8.8 **What is the effect of the time constant on discharging?**

Answer

Smaller values of the time constant RC lead to a more rapid discharge.

SQ 16.8.9 **Verify that an ohm times a farad is equivalent to a second.**

Proof

From Ohm's law

$V = IR$

and putting

$I = \frac{q}{t}$

gives

$V = \frac{qR}{t}$

so

$t = \frac{qR}{V}$

Since

$C = \frac{q}{V}$

therefore

$t = RC$

so the product of ohm and farad is the second.

TOPIC 9 Exponential Decay of Discharging Capacitor

SQ 16.9.1 **What does exponential mean?**

Definition

Exponential means a process whose rate of change at any instant is proportional to the present value of the quantity, so it changes quickly at first and then more slowly.

SQ 16.9.2 **Write the general exponential decay formula for a discharging capacitor.**

Formula

$x = x_0 e^{-t/RC}$

Here x_0 is the initial value of the quantity at

$t = 0$

and RC is the time constant.

SQ 16.9.3 **What is Euler's constant and what is its value?**

Answer

In the decay equation, e is a mathematical constant called Euler's constant, which is also used as the base of the natural logarithm. Its value is approximately 2.718.

SQ 16.9.4 **What does the negative exponent in the decay formula indicate?**

Answer

The negative exponent indicates that the quantity decreases exponentially over time.

SQ 16.9.5 **Write the decay equations for charge, voltage and current.**

Formulas

$Q = Q_0 e^{-t/RC}$
$V = V_0 e^{-t/RC}$
$I = I_0 e^{-t/RC}$

SQ 16.9.6 **Describe the shape of the decay graph.**

Answer

The graph starts with a steep slope, indicating fast discharge, and gradually flattens out as time progresses, indicating a slower discharge rate.

SQ 16.9.7 **Give the water tank analogy for the decay of charge.**

Analogy

The water level and pressure are analogous to charge and voltage, and the flow rate of water is analogous to the current.

Behaviour

When the level is high the flow is fast, and as the level comes down the rate of flow slows down due to the decreasing pressure.

SQ 16.9.8 **Define the time constant in terms of exponential decay.**

Derivation

Putting
$t = RC$
in
$Q = Q_0 e^{-t/RC}$
gives
$Q = Q_0 e^{-1}$

Definition

The time constant is the duration of time in which the initial charge Q_0 drops to about $0.37Q_0$, that is 37% of its initial value.

SQ 16.9.9 **What is the effect of a large or small time constant on circuit response?**

Answer

A smaller time constant means the circuit responds faster, while a larger time constant means it responds more slowly.

SQ 16.9.10 **Write the half-life of the charge on a discharging capacitor.**

Derivation

Putting
$Q = \frac{1}{2} Q_0$
in the decay equation and using natural logarithms.

Result

$t_{1/2} = 0.69\,RC$

SQ 16.9.11 **Which other physical phenomenon is described by exponential functions?**

Answer

Exponential functions frequently describe phenomena in physics such as radioactive decay, in which the rate of change in a process depends directly on its current value.

TOPIC 10 Use of Capacitors

SQ 16.10.1 **Why are capacitors used in refrigerators and air-conditioners?**

Reason

The compressors typically use single phase motors which can struggle to start on their own. A capacitor of high capacitance stores a big amount of electrical energy and releases it to the motor winding of the compressor.

SQ 16.10.2 **What is a start capacitor?**

Definition

A start capacitor acts like a “jump start” for the motor, providing a boost of power to get the motor rotating. Once the motor reaches a certain speed, a relay disconnects the capacitor as it is no longer needed.

SQ 16.10.3 **What is a run capacitor?**

Definition

Most refrigerators and air-conditioners also use a run capacitor which stays connected in the circuit to improve energy efficiency.

SQ 16.10.4 **How do capacitors form a time delay circuit?**

Answer

Capacitors in conjunction with resistors can form a time delay circuit. It is widely used in refrigerators and air conditioners to create a delay in triggering the power supply, as a precautionary measure to protect the devices from power fluctuation.

SQ 16.10.5 **Explain the working of a capacitor in a flashgun.**

Working

The capacitor accumulates charge from the battery of the camera. When the shutter button is pressed, the capacitor is connected to the flash tube.

Result

The rapid discharge provides the high current needed to ionize the xenon gas inside the tube, causing it to emit a burst of bright light.

SQ 16.10.6 **How are capacitors used as filters in rectifier circuits?**

Answer

Capacitors are widely used in rectifier circuits for the smoothing of pulsating DC into smooth DC.

SQ 16.10.7 **Give the medical uses of capacitors.**

Uses

Capacitors store the electrical energy needed to deliver a life-saving shock to the heart in case of cardiac arrest.
They are also used in imaging machines such as CT, MRI, X-ray and ultrasound, helping to filter out noise and improve image clarity, and they filter unwanted signals in EEG and ECG machines.