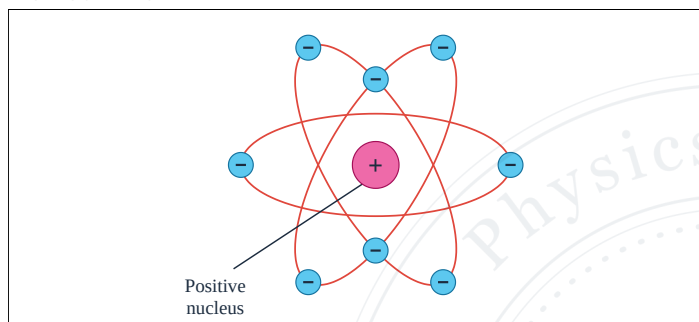


Topic 19.1 Mass Defect and Binding Energy**The Nucleus****Definition**

The nucleus is the dense, positively charged central part of an atom, of very small size, made of protons and neutrons and surrounded by electrons.

Discovery

Ernest Rutherford's experiments in 1911 indicated the existence of a dense, positively charged central part of the atom, the nucleus, of very small size and surrounded by electrons. In 1920 Rutherford suggested that the positive charge is due to protons inside the nucleus, and predicted another particle having no charge. The prediction came true when the neutron was discovered in 1932.

Do You Know

From alpha-particle scattering experiments, Lord Rutherford concluded that most of the atom is empty and that its mass is concentrated in a very small region called the nucleus.

Nuclear Physics

Nuclear physics is the study of the various aspects of atomic nuclei and sub-atomic particles: unstable nuclei giving off radiations, their decay, and nuclear reactions such as fission and fusion, which release huge amounts of energy. Radioactive isotopes are used as tracers in medical diagnostics and treatment, agriculture, scientific research and industry.

Nuclide

An atomic nucleus, often called a nuclide, is represented by the symbol ${}_Z^AX$.

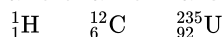
Symbols

- X is the chemical symbol of the element.
- Z is the atomic or charge number, the number of protons in the nucleus.
- A is the mass number, the total number of nucleons (protons and neutrons).

Number of Neutrons $N = A - Z$

Examples

Hydrogen, carbon and uranium are written as

**Size**

The nucleus is very dense, with a radius of the order of 10^{-14} m, surrounded by a cloud of electrons that gives the atom a radius of the order of 10^{-10} m.

LQ 1 starts here — mass defect, binding energy, and the information given by the curve of binding energy per nucleon against nucleon number

Mass Defect**Definition**

The mass of a nucleus is always less than the total mass of the protons and neutrons that make it up. The lost or missing mass is called the mass defect Δm

$$\Delta m = [Zm_p + (A - Z)m_n] - m_{\text{nucleus}}$$

Symbols

- m_p is the mass of a proton.
- m_n is the mass of a neutron.
- m_{nucleus} is the mass of the entire nucleus.

Binding Energy**Definition**

The energy required to break the nucleus into its constituents (neutrons and protons) is called binding energy. It is the potential energy of the nucleus, written B.E.

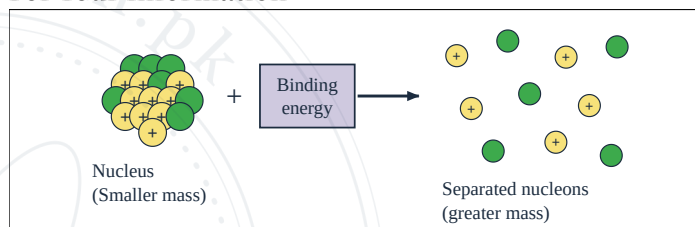
Mass into Energy

The missing mass is converted to energy, used in the formation of the nucleus. It is calculated from Einstein's mass-energy relation.

$$\Delta E = (\Delta m)c^2$$

Negative Value

Binding energy is considered to have a negative value, just as the absolute gravitational potential is taken as zero on the surface of the Earth.

For Your Information

Energy must be supplied to break the nucleus apart into its constituent protons and neutrons.

Binding Energy per Nucleon

A better measure is the binding energy per nucleon. It can be considered as the average energy needed to separate a nucleus into its individual nucleons.

Atomic Mass Unit

The masses of sub-atomic particles are usually given in atomic mass units (amu or u). One atomic mass unit is $\frac{1}{12}$ th of the mass of an unbound neutral atom of ${}^{12}_6\text{C}$

$$1 \text{ u} = 1.66 \times 10^{-24} \text{ g} = 1.66 \times 10^{-27} \text{ kg}$$

Some Atomic Masses

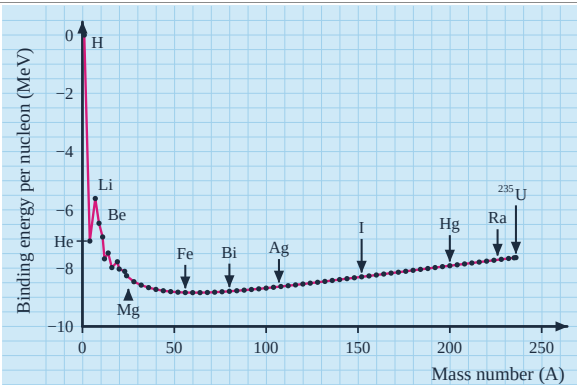
Particle	Mass (u)	Particle	Mass (u)
e	0.00055	${}^4\text{He}$	4.002603
n	1.008665	${}^7\text{Li}$	7.016004
${}^1\text{H}$	1.007825	${}^{10}\text{Be}$	10.013534
${}^2\text{H}$	2.014102	${}^{14}\text{N}$	14.0031
${}^3\text{H}$	3.01605	${}^{17}\text{O}$	16.9991
${}^3\text{He}$	3.01603		

Binding Energy Curve**Definition**

The binding energy curve is the graph of the binding energy per nucleon against the mass number A .

Use

A curve of binding energy per nucleon against the mass number A is useful. The position of a nucleus on this curve tells how stable it is and whether energy can be obtained from it by fission or by fusion.



From Hydrogen to Helium

The hydrogen nucleus, with just one proton, has zero binding energy. The fall from hydrogen to helium is large, which suggests a large energy release when hydrogen is converted to helium.

Most Stable Nuclide

The curve reaches a maximum binding energy per nucleon of about 8.7 MeV for iron ${}^{56}_{26}\text{Fe}$, the **most stable nuclide**. The region of most stable nuclei lies between mass numbers 50 and 80. After it the binding energy per nucleon decreases steadily to the end of the curve.

IQ 1 ends here

IQ 4 starts here — why very light and very heavy nuclei are unstable, and how they are useful as a source of a huge amount of energy

Light and Heavy Nuclei

Less Tightly Bound

The binding energy per nucleon is less for very light and very heavy nuclei, so their nucleons are not so tightly bound as those of the middle elements.

Fission

If a heavy nucleus is split into two nuclei lying near the lowest part of the curve, these are more tightly bound and stable. This process is called **nuclear fission**, and mankind already benefits from it in the production of huge amounts of power.

Fusion

The nucleons of any pair of neighbouring nuclei on the lighter side of the curve would be more tightly bound if the pair combined into a single nucleus. If deuterium and tritium fuse to form a more stable helium nucleus, a huge amount of energy is released. Such a process is not easy to initiate and needs extremely high temperature, such as occurs naturally in our Sun and the other stars.

Energy from Nuclear Reactions

Energy is obtained from any nuclear reaction in which the binding energy per nucleon of the products increases.

IQ 4 ends here

Book Examples

Example 19.1.1 Find the mass defect and binding energy of the helium nucleus, given that the mass of a proton is 1.672623×10^{-27} kg, the mass of a neutron is 1.674929×10^{-27} kg and the mass of the nucleus of ${}^4_2\text{He}$ is 6.646786×10^{-27} kg.

Given Data

$$m_p = 1.672623 \times 10^{-27} \text{ kg}$$

$$m_n = 1.674929 \times 10^{-27} \text{ kg}$$

$$m_{\text{nucleus}} = 6.646786 \times 10^{-27} \text{ kg}$$

To Find

Mass defect $\Delta m = ?$

Binding energy B. E = ?

Formula

$$\Delta m = 2m_p + 2m_n - m_{\text{nucleus}}$$

$$\text{B. E} = \Delta m c^2$$

Solution

Using the mass defect equation

$$\Delta m = 2(1.672623 \times 10^{-27}) + 2(1.674929 \times 10^{-27}) - 6.646786 \times 10^{-27}$$

$$= 0.048318 \times 10^{-27} \text{ kg}$$

Using Einstein's equation

$$\text{B. E} = 0.048318 \times 10^{-27} \text{ kg} \times (3.0 \times 10^8 \text{ m s}^{-1})^2$$

$$= 4.34862 \times 10^{-12} \text{ J}$$

Changing into eV

$$\text{B. E} = \frac{4.34862 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 27.2 \times 10^6 \text{ eV}$$

Result

$$\Delta m = 0.048318 \times 10^{-27} \text{ kg}$$

$$\text{B. E} = 27 \text{ MeV}$$

Short Questions

SQ 19.1.1 What is a nuclide? How is the number of neutrons found from its symbol?

Nuclide

An atomic nucleus, represented by the symbol ${}_Z^AX$, where Z is the number of protons and A the number of nucleons.

Number of neutrons

$$N = A - Z$$

For ${}^{235}_{92}\text{U}$ it is 143.

SQ 19.1.2 Define the atomic mass unit and give its value in kilograms.

Atomic mass unit

One atomic mass unit is $\frac{1}{12}$ th of the mass of an unbound neutral atom of carbon-12.

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

SQ 19.1.3 Why is binding energy considered to have a negative value?

Like gravitational potential

Energy has to be supplied to break the nucleus into separate nucleons, whose energy is taken as zero, so the bound nucleus has less energy than zero, just as absolute gravitational potential is negative when it is taken as zero far away.

SQ 19.1.4 Why is binding energy per nucleon a better measure of the stability of a nucleus than its total binding energy?

Average energy per nucleon

The binding energy per nucleon is the average energy needed to separate one nucleon from the nucleus, so it compares nuclei of different sizes on the same footing. The greater it is, the more tightly bound and stable the nucleus.

SQ 19.1.5 Which is the most stable nuclide? Give the reason.

Iron-56

${}^{56}_{26}\text{Fe}$ is the most stable nuclide because its binding energy per nucleon, about 8.7 MeV, is the maximum on the binding energy curve.

SQ 19.1.6 What does the large fall of the binding energy curve from hydrogen to helium suggest?

Large energy release

Hydrogen has zero binding energy per nucleon and helium about 7 MeV, so converting hydrogen to

helium should release a large amount of energy, which is what happens by fusion in the Sun.

MCQ 19.1.1 The number of neutrons in $^{235}_{92}\text{U}$ is:

- A 143 B 92
C 235 D 327

Solution: $N = A - Z = 235 - 92 = 143$.

MCQ 19.1.2 The radius of a nucleus is of the order of:

- A 10^{-10} m B 10^{-8} m
C 10^{-14} m D 10^{-18} m

Solution: The atom, with its electron cloud, is of the order of 10^{-10} m.

MCQ 19.1.3 One atomic mass unit is equal to:

- A 1.66×10^{-24} kg B 1.66×10^{-27} kg
C 1.67×10^{-31} kg D 9.1×10^{-31} kg

Solution: It is 1/12th of the mass of a carbon-12 atom, 1.66×10^{-24} g.

MCQ 19.1.4 The mass defect of a nucleus is:

- A $m_{\text{nucleus}} - Zm_p$ B $Zm_p + Am_n$
C $m_{\text{nucleus}} - [Zm_p + (A - Z)m_n]$ D $[Zm_p + (A - Z)m_n] - m_{\text{nucleus}}$

Solution: The nucleus is lighter than its separate protons and neutrons.

MCQ 19.1.5 A mass defect of 2.0×10^{-30} kg is equivalent to an energy of:

- A 6.0×10^{-22} J B 1.8×10^{-15} J
C 1.8×10^{-13} J D 3.6×10^{-13} J

Solution: $E = mc^2 = 2.0 \times 10^{-30} \times 9 \times 10^{16} = 1.8 \times 10^{-13}$ J.

MCQ 19.1.6 The binding energy per nucleon is maximum for:

- A helium B iron
C uranium D hydrogen

Solution: Iron-56 has about 8.7 MeV per nucleon.

MCQ 19.1.7 The region of the most stable nuclei lies between mass numbers:

- A 1 and 20 B 100 and 150
C 200 and 250 D 50 and 80

Solution: The curve is lowest, most tightly bound, between $A = 50$ and 80 .

MCQ 19.1.8 The binding energy of the hydrogen nucleus ^1_1H is:

- A zero B 2.2 MeV
C 7.1 MeV D 8.7 MeV

Solution: It has just one proton and nothing to bind.

MCQ 19.1.9 Energy is obtained from a nuclear reaction in which the binding energy per nucleon of the products:

- A decreases B increases
C remains the same D becomes zero

Solution: The products are more tightly bound than the reactants.

Topic 19.2 Nuclear Fission

LQ 2 starts here — nuclear fission, critical mass and the nuclear fission chain reaction

Nuclear Fission

Definition

A reaction in which a heavy nucleus splits up into two or more lighter nuclei of roughly equal size along with the emission of energy is called fission reaction.

Discovery

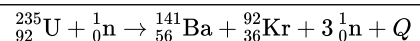
Otto Hahn and Fritz Strassmann of Germany observed that when slow moving neutrons are bombarded on $^{235}_{92}\text{U}$, the nuclear reaction gives $^{141}_{56}\text{Ba}$ and $^{92}_{36}\text{Kr}$, and on average about two or three neutrons are emitted. The

masses of both krypton and barium are less than the mass of uranium.

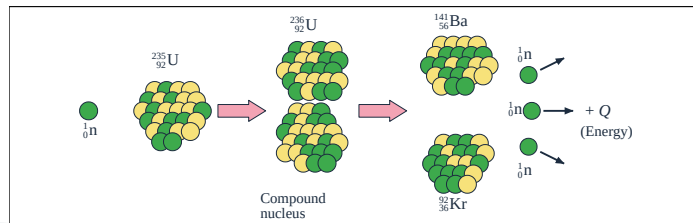
A Different Reaction

- The uranium nucleus breaks into two nuclei of almost equal size, whereas in other nuclear reactions the difference between the masses of the reactants and the products was not large.
- A very large amount of energy is given out.

Equation



Here Q is the energy given out. Comparing the total energy on the two sides shows that the fission of one uranium nucleus gives out about 200 MeV. The sums of the mass numbers and of the charge numbers are the same on both sides.



Explanation from the Curve

The binding energy per nucleon is greatest for the middle elements of the periodic table and a little less for the light or very heavy elements, whose nucleons are not so rigidly bound.

Energy per Fission

The binding energy per nucleon is about 7.6 MeV for uranium and about 8.5 MeV for barium and krypton, so the energy given out per nucleon is

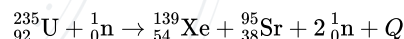
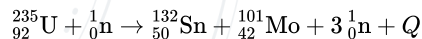
$$8.5 \text{ MeV} - 7.6 \text{ MeV} = 0.9 \text{ MeV}$$

For the fission of one uranium nucleus

$$235 \times 0.9 \text{ MeV} = 211.5 \text{ MeV}$$

Other Fragments

The fission of uranium does not always produce barium and krypton. Any two nuclei in the upper horizontal part of the binding energy curve could be produced.



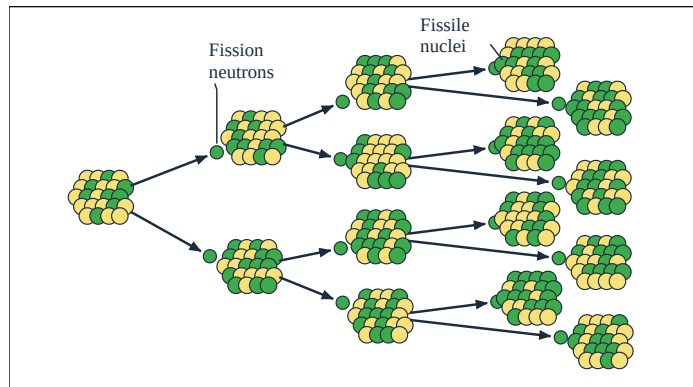
Fission Fragments

All these products, the fission fragments, are radioactive. Fission is possible in many other heavy elements, but it takes place very easily with slow neutrons in uranium-235 and plutonium-239, and these two are mostly used for fission.

Fission Chain Reaction

Definition

A nucleus of uranium-235 absorbs a neutron and breaks into two nuclei of almost equal masses, emitting two or three neutrons. By properly using these neutrons, fission can be produced in more uranium atoms, so that the reaction continuously maintains itself. This process is called a fission chain reaction.



Uncontrolled Chain Reaction

A slow neutron produces fission in one atom of a definite amount of $^{235}_{92}\text{U}$, and about three neutrons are emitted. If conditions are appropriate these produce fission in more atoms, and the process proceeds so rapidly that in an infinitesimally small time a large amount of energy is produced along with a huge explosion.

Controlled Chain Reaction

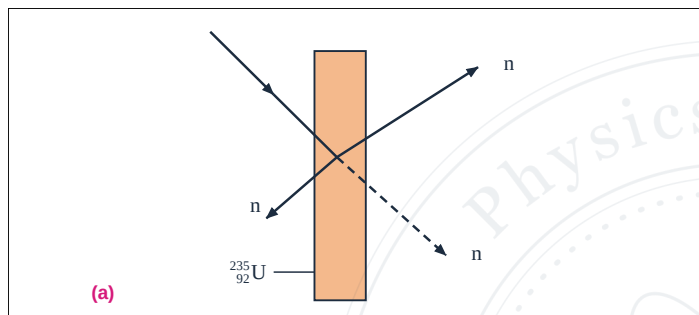
Conditions can be produced in which only one neutron out of all those created in one fission causes a further fission. The other neutrons escape or are absorbed in some medium other than uranium, and the chain reaction proceeds at its initial speed.

Conditions for a Chain Reaction

Whether a chain reaction is produced depends upon the shape and size of the fissile material.

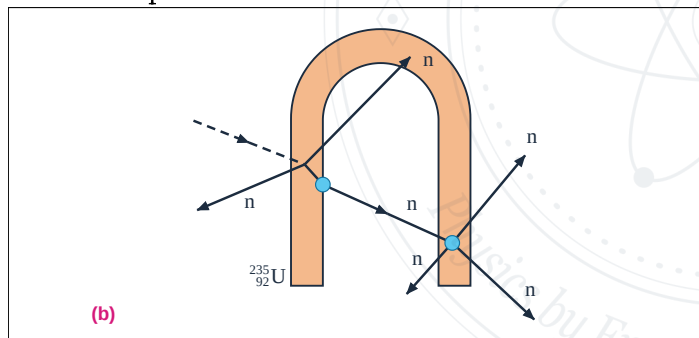
Thin Sheet

A fission reaction in a thin sheet of fissile material, such as enriched uranium-235. The resulting neutrons escape into the air, so they cannot produce a chain reaction.



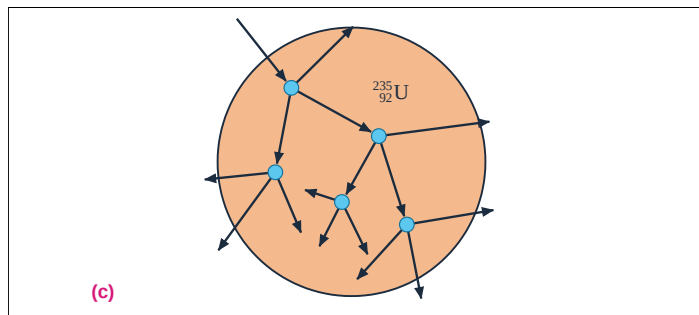
Favourable Conditions

Some of the neutrons produced in the first fission produce only one more fission, and here also no chain reaction is produced.



Spherical Lump

In a spherical lump of highly fissile material, mostly uranium-235, most of the neutrons produced by fission are absorbed before they escape out of the sphere if the lump is big enough, and they produce a chain reaction.



Critical Mass

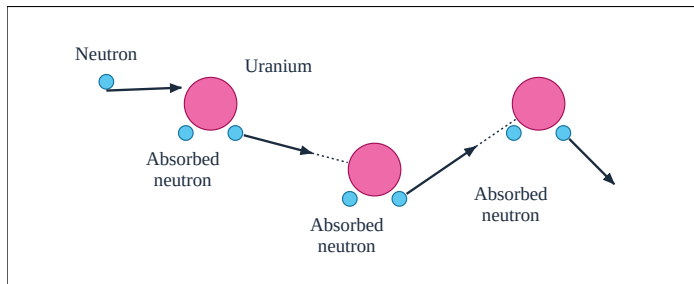
Definition

The minimum mass of fissile material like uranium-235 or plutonium-239 required to sustain a nuclear chain reaction is called critical mass. The volume of this mass of uranium is called the critical volume.

Mass and the Chain Reaction

- Much greater than the critical mass: the chain reaction proceeds at a rapid speed and a huge explosion or blast is produced. The atom bomb works on this principle.
- Less than the critical mass: the chain reaction does not proceed.
- Equal to the critical mass: the chain reaction proceeds at its initial speed and gives a source of energy. A nuclear reactor works on this principle, controlling the chain reaction to produce energy for human needs, as in nuclear power plants.

Do You Know



In a controlled chain reaction only one neutron, on average, from each fission event causes another nucleus to fission. As a result, energy is released at a steady rate.

Compared with Chemical Reactions

The energy released in a nuclear fission reaction is much more than in chemical reactions, such as the burning of fossil fuels.

LQ 2 ends here

Short Questions

SQ 19.2.1 In what two ways was the fission of uranium different from other nuclear reactions?

Two differences

- The nucleus breaks into two nuclei of almost equal size, whereas in other reactions the masses of reactants and products did not differ much.
- A very large amount of energy is given out.

SQ 19.2.2 How can the energy released in the fission of one uranium nucleus be estimated from the binding energy curve?

From binding energies

The binding energy per nucleon is about 7.6 MeV for uranium and 8.5 MeV for barium and krypton, so each nucleon gives out

$$8.5 \text{ MeV} - 7.6 \text{ MeV} = 0.9 \text{ MeV}$$

For 235 nucleons

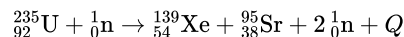
$$235 \times 0.9 \text{ MeV} = 211.5 \text{ MeV}$$

SQ 19.2.3 Does the fission of uranium-235 always give barium and krypton?

No, several pairs of fragments may be produced.

Other fragments

Any two nuclei in the upper horizontal part of the binding energy curve can be formed, such as tin and molybdenum or xenon and strontium.



SQ 19.2.4 Are fission fragments stable?

No, all the fission fragments are radioactive.

Why

They are formed with more neutrons than stable nuclei of the same elements, and they decay by giving off radiations.

SQ 19.2.5 Why is there no chain reaction in a thin sheet of uranium-235?

Neutrons escape

The neutrons produced by a fission in a thin sheet escape into the air before they meet another uranium nucleus, so they cannot produce further fission.

SQ 19.2.6 Define critical volume.

Critical volume

The volume of the critical mass of uranium, the minimum mass of fissile material that can sustain a nuclear chain reaction, is called the critical volume.

MCQ 19.2.1 The fission of uranium was discovered by:

- | | |
|----------------------------------|---------------------|
| A Otto Hahn and Fritz Strassmann | B Ernest Rutherford |
| C Albert Einstein | D Enrico Fermi |

Solution: They bombarded uranium-235 with slow neutrons.

MCQ 19.2.2 The energy given out in the fission of one uranium-235 nucleus is about:

- | | |
|------------|-----------|
| A 17.6 MeV | B 0.9 MeV |
| C 200 MeV | D 27 MeV |

Solution: About 0.9 MeV for each of 235 nucleons.

MCQ 19.2.3 In ${}_{92}^{235}\text{U} + {}_0^1\text{n} \rightarrow {}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + x {}_0^1\text{n}$ the value of x is:

- | | |
|-----|-----|
| A 1 | B 2 |
| C 4 | D 3 |

Solution: Mass numbers: $236 = 141 + 92 + x$, so $x = 3$.

MCQ 19.2.4 In ${}_{92}^{235}\text{U} + {}_0^1\text{n} \rightarrow {}_{54}^{139}\text{Xe} + {}_{38}^{94}\text{Sr} + 2 {}_0^1\text{n}$ the charge number Z of strontium is:

- | | |
|------|------|
| A 36 | B 40 |
| C 92 | D 38 |

Solution: Charge numbers: $92 = 54 + Z$, so $Z = 38$.

MCQ 19.2.5 If the binding energy per nucleon is 7.6 MeV for uranium and 8.5 MeV for its fragments, the energy released per nucleon in fission is:

- | | |
|-----------|-------------|
| A 0.9 MeV | B 16.1 MeV |
| C 7.6 MeV | D 211.5 MeV |

Solution: $8.5 - 7.6 = 0.9$ MeV.

MCQ 19.2.6 Fission takes place very easily with slow neutrons in:

- | | |
|---------------------------------|-------------------------|
| A uranium-238 and thorium-232 | B iron-56 and nickel-62 |
| C uranium-235 and plutonium-239 | D deuterium and tritium |

Solution: These two are mostly used for fission.

MCQ 19.2.7 If the mass of uranium is less than the critical mass, the chain reaction:

- | | |
|---------------------------------|-------------------------|
| A proceeds rapidly | B does not proceed |
| C proceeds at its initial speed | D produces an explosion |

Solution: Too many neutrons escape for each fission to cause another.

MCQ 19.2.8 An atom bomb works on:

- | | |
|-------------------------------|----------------------------------|
| A a controlled chain reaction | B an uncontrolled chain reaction |
| C nuclear fusion only | D radioactive decay |

Solution: Its mass of uranium is much greater than the critical mass.

Topic 19.3 Nuclear Reactor

LQ 3 starts here — the nuclear power reactor with its components and their functions

Nuclear Power Plant

Definition

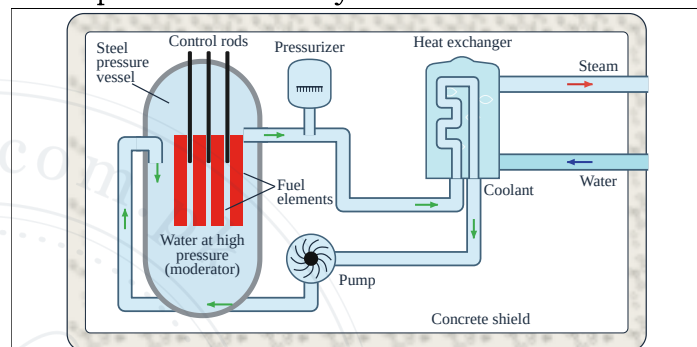
A nuclear reactor is the part of a nuclear power station in which a controlled fission chain reaction produces heat, playing the same part as the furnace of a thermal power station.

Reactor and Furnace

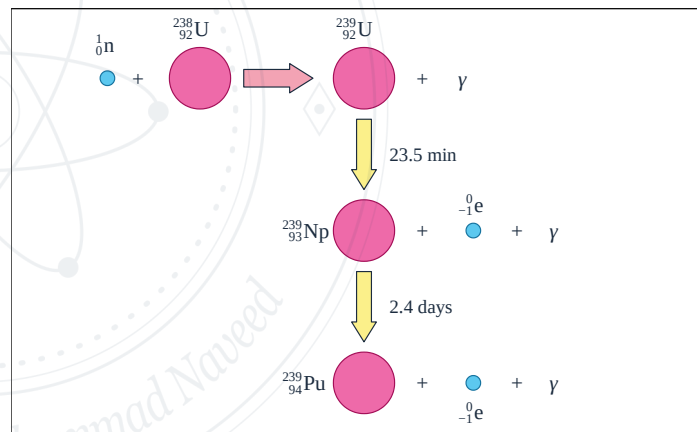
In a nuclear power station the reactor plays the same part as the furnace in a thermal power station. In a furnace coal or oil is burnt to produce heat, while in a reactor a fission reaction produces heat.

From Fission to Electricity

The fission of an atom of uranium or any other heavy atom produces about 200 MeV. This energy appears as the kinetic energy of the fission fragments. The fast fragments collide with one another and with the uranium atoms, so their kinetic energy is transformed into heat. The heat produces steam, the steam rotates the turbine and the turbine rotates the generator, which produces electricity.



Do You Know



An induced nuclear reaction in which ${}_{92}^{238}\text{U}$ is transmuted into the transuranium element plutonium ${}_{94}^{239}\text{Pu}$, with half-life 24,100 years.

Parts of a Reactor

Four Parts

A reactor usually has four important parts.

Core

The most important and vital part of a reactor is the core, where the fuel is kept in the shape of cylindrical tubes. Uranium was the fuel of the first generation reactors.

Enriched Fuel

Naturally occurring uranium contains only 0.7 percent of ${}_{92}^{235}\text{U}$, so in reactor fuel the quantity of ${}_{92}^{235}\text{U}$ is enriched to 2.4 to 3.0 percent. Nowadays plutonium-239 and uranium-233 are also used as fuel.

Moderator

The fuel rods are placed in a substance of small atomic weight, such as water or heavy water, called the moderator. It slows down the neutrons produced in fission and directs them towards the fuel. The fission neutrons are very fast and energetic and are

not suitable for producing fission in fuel like $^{235}_{92}\text{U}$ or $^{239}_{94}\text{Pu}$; slow neutrons are more useful. Heavy water is made of ^2_1H , a heavy isotope of hydrogen, instead of ^1_1H .

Control Rods

The number of neutrons is controlled so that only one neutron of all those produced in a fission produces a further fission. This is done by cadmium or boron rods, which absorb neutrons. The control rods are moved in or out of the core to control the neutrons that can start further fission, and so the speed of the chain reaction is kept under control. In an emergency or for repair, the control rods are allowed to fall back into the reactor, which stops the chain reaction and shuts down the reactor.

Coolant and Heat Exchanger

The chain reaction heats the core to about 500°C . This heat is carried to the heat exchanger by water, heavy water or any other liquid under high pressure. In the heat exchanger it produces high temperature steam from ordinary water, and the steam runs the turbine, which rotates the generator to produce electricity.

Cooling the Steam

The steam coming out of the turbine is at up to 50°C . It is cooled back into water, generally with water from a river or the sea.

Nuclear Power in Pakistan

KANUPP

In the Karachi Nuclear Power Plant (KANUPP), heavy water is used as the moderator and also to carry the heat from the core to the heat exchanger. Sea water cools the steam coming out of the turbine.

Other Plants

Three nuclear power plants are operational in Karachi and four at Chashma (Mianwali), with a total output capacity of more than 3,000 MW. The Chashma plants are cooled with water from the nearby Indus river.

Do You Know



This symbol is universally used to indicate an area where radioactivity is being handled or artificial radiations are being produced.

Nuclear Waste

Definition

The used fuel removed from a reactor, which still holds intense radioactive substances, is the nuclear waste.

Spent Fuel

The fuel once loaded can run the reactor for a few months, after which the fissile material decreases and the used fuel is replaced with fresh fuel. Intense radioactive substances remain in the used fuel, with half-lives of many thousand years, and their radiations and particles are very injurious to living things.

Disposal

There is no proper arrangement for the disposal of nuclear waste. It cannot be dumped into the oceans or left anywhere it would contaminate the environment through the soil or the air, and it must not get into drinking water. The best place found so far is the bottom of old salt mines, very dry and thousands of

metres below the Earth's surface, where the waste can decay without polluting the surface.

Do You Know

Radioactive wastes are of three types: high level, medium level and low level. All of them are dangerous for ground water and the land environment.

For Your Information

Radioactive waste is very difficult to dispose of safely because of its long half-lives. The half-life of Pu is 24,100 years, so it remains dangerous for about 1,82,000 years.

LQ 3 ends here

Short Questions

SQ 19.3.1 What part does the reactor play in a nuclear power station?

Like a furnace

The reactor plays the same part as the furnace of a thermal power station: in a furnace coal or oil is burnt to give heat, while in the reactor fission gives heat.

SQ 19.3.2 What is heavy water? Why is it used in a reactor?

Heavy water

Water made of ^2_1H , the heavy isotope of hydrogen, instead of ^1_1H .

Moderator

It is a substance of small atomic weight, so it slows down the fast fission neutrons, which are then more useful for producing fission.

SQ 19.3.3 How is a nuclear reactor shut down in an emergency?

Control rods

The cadmium or boron control rods are allowed to fall back into the reactor. They absorb the neutrons, the chain reaction stops and the reactor shuts down.

SQ 19.3.4 Why must nuclear waste not be dumped into the oceans? Where is it best stored?

Long-lived radioactivity

The used fuel holds intense radioactive substances with half-lives of many thousand years, which would contaminate the environment and drinking water.

Salt mines

The best place found so far is the bottom of old salt mines, very dry and thousands of metres below the surface.

SQ 19.3.5 What is used as the moderator and the coolant in KANUPP?

Heavy water

In the Karachi Nuclear Power Plant heavy water is the moderator and also carries the heat from the core to the heat exchanger.

Sea water

Sea water cools the steam coming out of the turbine.

MCQ 19.3.1 The temperature of the core of a nuclear reactor rises to about:

A 50°C	B 15 million K
C 500°C	D 100°C

Solution: The steam leaving the turbine is at up to 50°C .

MCQ 19.3.2 The percentage of uranium-235 in naturally occurring uranium is only:

A 0.7 percent	B 2.4 percent
C 3.0 percent	D 7 percent

Solution: Reactor fuel is enriched to 2.4 to 3.0 percent.

MCQ 19.3.3 In the fuel of first generation reactors uranium-235 is enriched to:

- A 0.7 percent B 50 percent
C 90 to 100 percent D **2.4 to 3.0 percent**

Solution: Natural uranium holds only 0.7 percent.**MCQ 19.3.4 Plutonium-239 is produced from uranium-238 through:**

- A alpha decay B fusion with deuterium
C **neutron capture and two beta decays** D gamma decay only

Solution: U-238 + n gives U-239, which decays to Np-239 and then to Pu-239.**MCQ 19.3.5 The half-life of plutonium-239 is:**

- A 23.5 minutes B 2.4 days
C 1620 years D **24,100 years**

Solution: So it remains dangerous for about 1,82,000 years.**MCQ 19.3.6 The total output capacity of Pakistan's nuclear power plants is more than:**

- A 300 MW B **3,000 MW**
C 30,000 MW D 30 MW

Solution: Three plants in Karachi and four at Chashma.**Topic 19.4 Nuclear Fusion****Nuclear Fusion****Definition**A nuclear reaction in which two light nuclei merge to form a heavy nucleus is called fusion reaction.**Energy from Light Nuclei**

The curve of Fig. 19.1 shows that the binding energy per nucleon increases up to A of 50. So when two light nuclei merge to form a heavier nucleus whose mass number is less than 50, energy is given out. When two protons and two neutrons merge to form a helium nucleus, about 27 MeV is given out.

Mass and Energy

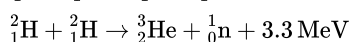
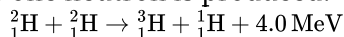
In a fusion reaction some mass is lost and its equivalent energy is given out. More energy per nucleon can be obtained from fusion than from fission.

Why Fusion Is Difficult

Two positively charged nuclei must be brought very close to one another, and work has to be done against the electrostatic force of repulsion between them. A very large amount of energy must be spent to start a fusion reaction, even though a greater amount is obtained from it.

Fusion Reactions**Deuterium Reactions**

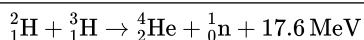
The fusion of two light nuclei is most probable where one proton or one neutron is produced.



In both reactions about 1.0 MeV per nucleon is produced, equal to the energy produced in fission.

Deuterium and Tritium

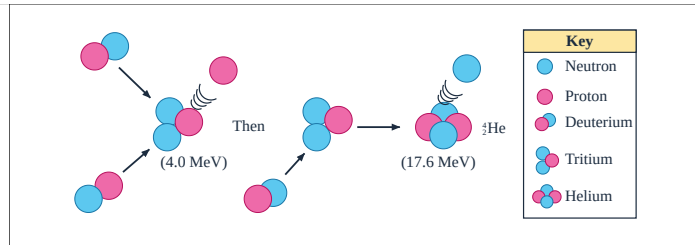
If ${}^2_1\text{H}$ and ${}^3_1\text{H}$ are forced to fuse, 17.6 MeV is obtained.



Deuterium and tritium are termed fusion fuels.

Producing Fusion**Accelerating the Nuclei**

To overcome the repulsion, the two nuclei are accelerated towards one another at a very high speed. This method has been used in research on the fusion of ${}^2_1\text{H}$ and ${}^3_1\text{H}$, but it cannot be used on a large scale.

**Thermonuclear Reaction**

The speed of the atoms of a substance increases with its temperature. The temperature at which light nuclei reach the speed needed to start fusion is about 15 million kelvin, and a reaction at such a temperature is called a thermonuclear reaction. Ordinarily such a temperature cannot be achieved, but it is reached for a very short time in the explosion of an atom bomb.

Hydrogen Bomb

Fusion has so far been observed in the testing of the hydrogen bomb, triggered by an atomic bomb. It has not yet been brought under control like fission, so it is not used to produce electricity, though efforts are in full swing to find a way.

Sun and Stars

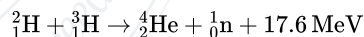
The fusion process is taking place in the core of stars, including our Sun.

For Your Information

An artificial Sun has been designed to mimic the real Sun. The Experimental Advanced Superconducting Tokamak (EAST) project in China uses powerful magnetic fields to confine superheated plasma at over 100 million °C, in an attempt to fuse hydrogen isotopes into helium nuclei and release clean energy, just like our Sun.

Short Questions**SQ 19.4.1 What is a thermonuclear reaction?****Thermonuclear reaction**

A fusion reaction started by raising the temperature, at about 15 million kelvin, so that the light nuclei move fast enough to fuse.

SQ 19.4.2 Write the fusion of deuterium and tritium and the energy it releases.**Deuterium-tritium fusion**

It releases 17.6 MeV.

SQ 19.4.3 Why is fusion not yet used to produce electricity?**Not controlled**

Fusion has not been brought under control like fission; it has been observed only in the testing of the hydrogen bomb.

SQ 19.4.4 How is the very high temperature for fusion reached in a hydrogen bomb?**Atomic bomb trigger**

The hydrogen bomb is triggered by an atomic bomb, whose explosion gives the required temperature for a very short time.

MCQ 19.4.1 The temperature needed to start a thermonuclear reaction is about:

- A **15 million kelvin** B 500 °C
C 15,000 kelvin D 1 million °C

Solution: Only then do the light nuclei move fast enough to fuse.**MCQ 19.4.2 The energy released when deuterium and tritium fuse is:**

- A 4.0 MeV B 3.3 MeV
C 200 MeV D **17.6 MeV**

Solution: ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + 17.6 \text{ MeV}$.

MCQ 19.4.3 In ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$ the energy released is:

- A 4.0 MeV
B 3.3 MeV
C 17.6 MeV
D 27 MeV

Solution: The other deuterium reaction, giving tritium and a proton, releases 4.0 MeV.

MCQ 19.4.4 Fusion gives out energy when the mass number of the product nucleus is less than:

- A 4
B 235
C 50
D 150

Solution: The binding energy per nucleon increases up to $A = 50$.

MCQ 19.4.5 The experimental advanced superconducting Tokamak (EAST) is a project of:

- A China
B Pakistan
C Switzerland
D Germany

Solution: It is an artificial Sun, confining plasma with magnetic fields.

Topic 19.5 Activity and Half-Life

Radioactive Decay

Definition

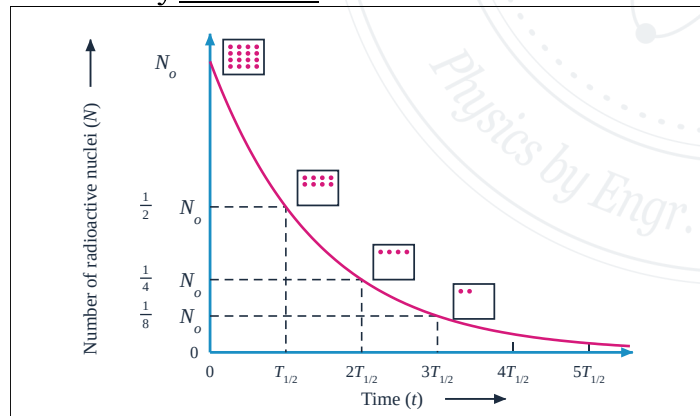
Radioactive decay is the process in which an unstable nucleus emits an α or β -particle and is transformed into some other element.

Random Decay

Whenever an α or β -particle is emitted from a radioactive element, it is transformed into some other element. This decay is quite random and does not follow any regular pattern; fluctuation in decay curves is also evidence of its random nature. We cannot foretell when any particular atom will decay: it could decay immediately or remain unchanged for thousands of years.

Decay Curve

Though a single nucleus cannot be predicted, a very large sample decays with a consistent pattern when the number of radioactive nuclei is plotted against time. The curve never touches zero, which means that radioactivity never ends.



Activity

Definition

The activity of a radioactive substance is the number of particles it emits per second. The number of emissions per second is called the decay rate or activity, and its SI unit is the becquerel (Bq).

$$1 \text{ Bq} = 1 \text{ decay per second}$$

Count Rate

The count rate of a nuclear radiation detector is the number of counts recorded per second. It is directly proportional to the activity. As time goes on, the activity of a source decreases in a consistent manner.

LQ 6 starts here — half-life, the relation between the number of half-lives and time, and the remaining and decayed quantity after

several half-lives

Half-Life

Definition

The time taken for the activity of a sample to decrease to half of any starting value is called half-life.

Two Conclusions

- No radioactive element can completely decay: in any half-life only half of the nuclei decay, so an infinite time is required for all the atoms to decay.
- The number of atoms decaying in a particular period is proportional to the number present at the beginning of that period.

Decay Constant

Definition

The decay constant λ of a radioactive element is the fraction of its atoms decaying per unit time.

Decay Equation

In an interval Δt the number of decaying atoms ΔN is proportional to the interval and to the number of atoms N

$$\Delta N \propto -N\Delta t$$

$$\Delta N = -\lambda N\Delta t$$

Here λ is the constant of proportionality, called the decay constant.

Meaning

If the decay constant is large, more atoms decay in a given interval, and if it is small fewer decay. The decay constant is the fraction of the decaying atoms per unit time, and its unit is s^{-1} .

Activity and Decay Constant

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

As $\Delta N/\Delta t$ is the activity

$$A = \lambda N$$

This is the absolute activity, in which the number of atoms N decreases with time.

Number Left and Decayed

Number Left

At first the sample holds N_0 atoms. Half of the atoms decay in each half-life.

After one half-life

$$\frac{N_0}{2}$$

After two half-lives

$$\frac{1}{2} \times \frac{N_0}{2} = \left(\frac{1}{2}\right)^2 N_0$$

After three half-lives

$$\frac{1}{2} \times \left(\frac{1}{2}\right)^2 N_0 = \left(\frac{1}{2}\right)^3 N_0$$

After n half-lives

$$N = \left(\frac{1}{2}\right)^n N_0$$

Half-Lives and Time

In a time t the number of half-lives is

$$n = \frac{t}{T_{1/2}}$$

Number Decayed

The number of decayed nuclides is the original number less the number left.

After the first half-life

$$N = N_0 - \frac{N_0}{2} = \frac{N_0}{2}$$

After the second half-life

$$N = N_0 - \frac{N_0}{4} = \frac{3N_0}{4}$$

After the third half-life

$$N = N_o - \frac{N_o}{8} = \frac{7N_o}{8}$$

After n half-lives

$$N = N_o \left(1 - \frac{1}{2^n}\right)$$

Different Half-Lives

Every radioactive element decays according to the same curve, but each has its own half-life. The half-life of uranium-238 is 4.5×10^9 years and that of radium-226 is 1620 years, while that of radon gas is only 3.8 days and that of uranium-239 23.5 minutes.

Exponential Decay

Statement

The number of undecayed nuclei of a radioactive element decreases exponentially with time.

Decay Law

The decay of a radioactive element can be found from its half-life or its decay constant. With calculus the number of undecayed nuclei N after a time t is

$$N = N_o e^{-\lambda t}$$

Here N_o is the initial number of nuclides.

Half-Life and Decay Constant

When N reduces to $N_o/2$ the time t is the half-life $T_{1/2}$

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2$$

$$\lambda T_{1/2} = \ln 2 = 0.693$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

So if the decay constant of a radioactive element is known, its half-life can be found.

IQ 6 ends here

Artificial Radioactivity

Definition

Any stable element can be made radioactive by bombarding it with very high energy particles. The bombardment excites the nuclei, which become unstable and radioactive. Such elements are called artificial radioactive elements.

Book Examples

Example 19.5.1 The half-life of polonium ($^{210}_{84}\text{Po}$) is 140 days. If there are 1,000 atoms of polonium initially present in a sample, how many of them will decay in 280 days?

Given Data

$$N_o = 1000$$

$$T_{1/2} = 140 \text{ days}$$

$$t = 280 \text{ days}$$

To Find

Atoms decayed in 280 days = ?

Formula

Half of the atoms present decay in each half-life.

Solution

Number of atoms decaying in the first 140 days

$$\frac{1000}{2} = 500$$

Number of polonium atoms left

$$1000 - 500 = 500$$

Number of atoms decaying in the next 140 days

$$\frac{500}{2} = 250$$

Total atoms decaying in 280 days

$$500 + 250 = 750$$

Result

750 atoms

Example 19.5.2 Americium-241 is an artificially produced radioactive element that emits α -particles. Its sample of mass 5.1 μg is found to have an activity $5.9 \times 10^5 \text{ Bq}$. Determine, for this sample: (i) total number of nuclei (ii) decay constant (iii) half-life in years.

Given Data

$$m = 5.1 \mu\text{g} = 5.1 \times 10^{-6} \text{ g}$$

$$M = 241 \text{ g mol}^{-1}$$

$$A = 5.9 \times 10^5 \text{ Bq}$$

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

To Find

(i) Total number of nuclei $N = ?$

(ii) Decay constant $\lambda = ?$

(iii) Half-life $T_{1/2} = ?$

Formula

$$N = \frac{mN_A}{M}$$

$$A = \lambda N$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

Solution

(i) Using Avogadro's number

$$N = \frac{5.1 \times 10^{-6} \text{ g} \times 6.02 \times 10^{23}}{241 \text{ g}} = 1.27 \times 10^{16}$$

(ii) Using the activity equation

$$5.9 \times 10^5 \text{ s}^{-1} = \lambda \times 1.27 \times 10^{16}$$

$$\lambda = \frac{5.9 \times 10^5}{1.27 \times 10^{16}} = 4.65 \times 10^{-11} \text{ s}^{-1}$$

(iii) Half-life

$$T_{1/2} = \frac{0.693}{4.65 \times 10^{-11} \text{ s}^{-1}} = 1.49 \times 10^{10} \text{ s}$$

Converting into years

$$T_{1/2} = \frac{1.49 \times 10^{10}}{365 \times 24 \times 60 \times 60} \text{ years} = 472 \text{ years}$$

Result

$$N = 1.27 \times 10^{16}$$

$$\lambda = 4.6 \times 10^{-11} \text{ s}^{-1}$$

$$T_{1/2} = 472 \text{ years}$$

Short Questions

SQ 19.5.1 What shows that radioactive decay is random?

Random decay

We cannot foretell when any particular atom will decay: it may decay at once or remain for thousands of years, and the fluctuation in decay curves is evidence of this random nature.

SQ 19.5.2 Define the decay constant and give its unit.

Decay constant

The fraction of the decaying atoms per unit time, the constant λ in the decay equation. Its unit is s^{-1}

$$\Delta N = -\lambda N \Delta t$$

SQ 19.5.3 What is the count rate? How is it related to the activity?

Count rate

The number of counts recorded per second by a nuclear radiation detector. It is directly proportional to the activity of the source.

SQ 19.5.4 State the exponential law of radioactive decay.

Exponential decay

The number of undecayed nuclei falls exponentially with time.

$$N = N_0 e^{-\lambda t}$$

Here N_0 is the initial number of nuclides and N the number undecayed after time t .

SQ 19.5.5 What are artificial radioactive elements?

Artificial radioactivity

Stable elements made radioactive by bombarding them with very high energy particles, which excite the nuclei and make them unstable.

MCQ 19.5.1 The SI unit of activity is:

- | | |
|--------------------|----------------------|
| A second | B per second squared |
| C curie per second | D becquerel |

Solution: 1 Bq = 1 decay per second.

MCQ 19.5.2 The unit of the decay constant is:

- | | |
|------------|--------------|
| A s | B Bq s |
| C s^{-1} | D $m s^{-1}$ |

Solution: It is the fraction of atoms decaying per unit time.

MCQ 19.5.3 A nuclide has a decay constant of $6.93 \times 10^{-3} s^{-1}$. Its half-life is:

- | | |
|-----------|----------|
| A 6.93 s | B 100 s |
| C 0.693 s | D 1000 s |

Solution: $T = 0.693/\lambda = 0.693/6.93 \times 10^{-3} = 100$ s.

MCQ 19.5.4 A sample of 2×10^{20} nuclei has a decay constant of $1 \times 10^{-10} s^{-1}$. Its activity is:

- | | |
|-------------------------|--------------------------|
| A 2×10^{10} Bq | B 2×10^{30} Bq |
| C 5×10^{29} Bq | D 2×10^{-10} Bq |

Solution: $A = \lambda N = 1 \times 10^{-10} \times 2 \times 10^{20} = 2 \times 10^{10}$ Bq.

MCQ 19.5.5 After three half-lives, the fraction of a radioactive sample that has decayed is:

- | | |
|-------|-------|
| A 1/8 | B 3/4 |
| C 7/8 | D 1/3 |

Solution: $N = N_0(1 - 1/2^3) = 7N_0/8$.

MCQ 19.5.6 A 64 g sample has a half-life of 5 days. The mass left after 20 days is:

- | | |
|-------|--------|
| A 4 g | B 16 g |
| C 8 g | D 2 g |

Solution: 20 days are 4 half-lives: $64/16 = 4$ g.

MCQ 19.5.7 The half-life of radon gas is:

- | | |
|---------------------------|----------------|
| A 1620 years | B 23.5 minutes |
| C 4.5×10^9 years | D 3.8 days |

Solution: Radium-226 is 1620 years and uranium-238 is 4.5×10^9 years.

MCQ 19.5.8 In $N = N_0 e^{-\lambda t}$ the symbol N stands for the:

- | | |
|------------------------------|-----------------------------------|
| A nuclei decayed in time t | B undecayed nuclei after time t |
| C initial number of nuclides | D activity at the start |

Solution: N_0 is the initial number of nuclides.

MCQ 19.5.9 The time taken for the activity of a sample to fall to half is the:

- | | |
|-------------------|-----------------|
| A decay constant | B half-life |
| C mean count rate | D critical time |

Solution: $T = 0.693/\lambda$.

Topic 19.6 Radioactive Tracers

LQ 5 starts here — radioactive tracers and their applications in medical diagnostics and treatment

Radioactive Tracers

Definition

A radioactive isotope behaves in just the same way as the normal isotope inside a living organism, but its location and concentration are easily found by measuring the radiation it emits. It therefore acts as an indicator or tracer that makes it possible to follow the course of a chemical or biological process.

Technique

Radioactive atoms are substituted for stable atoms of the same kind in a substance, and the tagged atoms are followed with a radiation detector.

In the Body

Some chemicals, such as the hydrogen and sodium in water and food, are distributed uniformly throughout the human body. Certain other chemicals are selectively absorbed by certain organs.

Making Radioisotopes

Radioisotopes of many elements are easily made by bombardment with neutrons and other particles. Being available and inexpensive, their use in medicine, agriculture, scientific research and industry has expanded tremendously.

Uses of Tracers

In Research

Radioisotopes show what happens in many complex chemical reactions and how they proceed. In biology they help investigate the chemical reactions in plants and animals.

In Agriculture

A small amount of radioisotope mixed with fertilizer shows with a radiation detector how much fertilizer a plant takes up, so farmers know the proper amount to use. Radiation-induced mutations have developed improved varieties of rice, chickpea, wheat and cotton, with better plant structure, more resistance to diseases and pests, and better yield and grain quality. The Nuclear Institute for Agriculture and Biology (NIAB) at Faisalabad is doing a great job in this regard.

Uses in Medical Diagnostic and Treatment of Diseases

Study of the Body

Tracers are widely used in medicine to study the process of digestion and the way chemical substances move about in the human body.

Tracers in Organs

Radio-iodine is absorbed mostly by the thyroid gland, phosphorus by bones and cobalt by the liver, so they can serve as tracers. A small quantity of a low-activity radioisotope mixed with the stable isotope is given to a patient by injection or otherwise, and its location in diseased tissue is found with radiation detectors.

Thyroid and Blood

Radioactive iodine is used to check that a person's thyroid gland is working properly: a diseased or hyperactive gland absorbs more than twice the amount of a normal gland. Radioactive iodine-131 is also used to combat cancer of the thyroid gland. A similar method with the radioactive isotope sodium-24 studies the circulation of blood.

Cancer Cells

Cells that multiply rapidly absorb more radiation and are more easily destroyed than normal cells by ionizing radiation.

Imaging and Seeds

Radioactive tracers in imaging devices have helped in the understanding, diagnosis and treatment of many diseases. In some cases radioisotopes in capsules, known as seeds, are implanted in the malignant tissue for local and short ranged treatment. For skin cancer phosphorus-32 or strontium-90 may be used, but the dose has to be carefully controlled to avoid harming healthy tissues.

Side Effects

A patient undergoing radiation treatment often feels ill, as the radiation also damages some healthy tissues.

Some Radioisotopes and their Uses

Isotope	Half-life	Example of use
Sodium ^{24}Na	15 hours	Plasma volume
Iron ^{52}Fe	8.3 hours	Imaging bone marrow function
Technetium ^{99}Tc	6 hours	Thyroid uptake scans
Iodine ^{131}I	8.3 days	Kidney tests
Iodine ^{125}I	60 days	Plasma volume Vein flow

LQ 5 ends here

Short Questions

SQ 19.6.1 What are tagged atoms?

Tagged atoms

Radioactive atoms substituted for stable atoms of the same kind in a substance. Their radiation lets a radiation detector follow them through a chemical or biological process.

SQ 19.6.2 How do radioisotopes help a farmer use fertilizer?

Fertilizer uptake

A small amount of radioisotope is mixed with the fertilizer and a radiation detector measures how much the plant takes up, so the farmer knows the proper amount to use.

SQ 19.6.3 How have radiation-induced mutations improved crops?

Improved varieties

They have developed improved varieties of rice, chickpea, wheat and cotton, with better plant structure, more resistance to diseases and pests, and better yield and grain quality.

SQ 19.6.4 Why has the use of radioisotopes expanded so much?

Easily made

Radioisotopes of many elements are easily made by bombardment with neutrons and other particles, so they are available and inexpensive.

MCQ 19.6.1 Radio-iodine is absorbed mostly by the:

- | | |
|-----------------|-----------|
| A liver | B bones |
| C thyroid gland | D kidneys |

Solution: Phosphorus goes to bones and cobalt to the liver.

MCQ 19.6.2 The circulation of blood is studied with radioactive:

- | | |
|-----------------|--------------|
| A sodium-24 | B iodine-131 |
| C phosphorus-32 | D cobalt-60 |

Solution: Iodine-131 combats thyroid cancer.

MCQ 19.6.3 For skin cancer the radioisotope used may be:

- | | |
|-------------|---------------------------------|
| A sodium-24 | B iodine-125 |
| C iron-52 | D phosphorus-32 or strontium-90 |

Solution: The dose is carefully controlled to spare healthy tissue.

MCQ 19.6.4 Cobalt used as a tracer is absorbed mostly by the:

- | | |
|-----------------|---------|
| A thyroid gland | B bones |
| C blood plasma | D liver |

Solution: Radio-iodine goes to the thyroid and phosphorus to bones.

MCQ 19.6.5 The half-life of technetium-99 used for thyroid uptake scans is:

- | | |
|------------|-----------|
| A 15 hours | B 6 hours |
| C 8.3 days | D 60 days |

Solution: Sodium-24 is 15 hours and iodine-131 8.3 days.

Topic 19.7 Annihilation Reactions

Annihilation

Definition

Annihilation is an event that takes place whenever a particle and its antiparticle come close to each other. The annihilation of a positron-electron pair was discussed in the previous class.

High Energy Collisions

Very High Energies

Research on elementary particles needs very high energies. Instead of colliding at their rest mass energies, the particles are accelerated to nearly the speed of light and then collided.

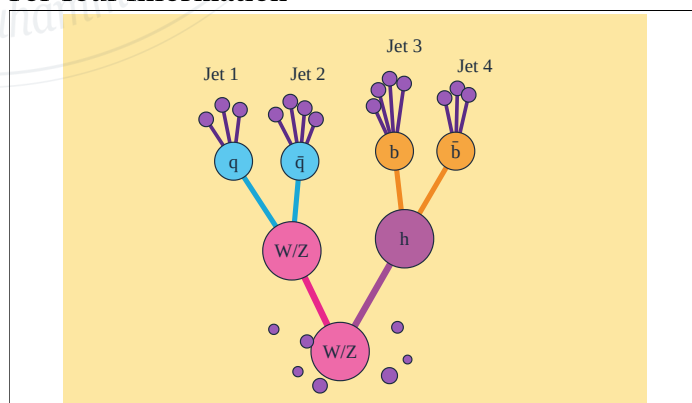
Large Hadron Collider

At CERN's Large Hadron Collider (LHC) the proton p^+ the antiproton $\bar{p}$ and other heavy nuclei are collided at nearly the speed of light to create conditions similar to the early universe. Their collisions at energies of the order of TeV offer insight into the conditions of the early universe.

Discoveries

In such collisions the electro-weak force mediation particles, the W^+ and W^- and the Z bosons, were found as predicted, along with the top and bottom quarks and, most significantly, the Higgs boson in July 2012.

For Your Information



A schematic diagram of a p^+ and $\bar{p}$ collision. An image of a 7 TeV proton-proton collision in CMS shows more than 100 charged particles produced.

Electron-Positron Annihilation

Mass into Energy

When an electron and a positron annihilate, their mass is converted into the energy of gamma-ray photons.

$$E = 2mc^2$$

Two gamma-ray photons are emitted, each of 0.51 MeV, travelling in opposite directions so that momentum is conserved.

Book Examples

Example 19.7.1 Calculate the energy of the gamma ray photons emitted during the annihilation of an electron-positron pair.

Given Data

$$m_{e^+} = 9.1 \times 10^{-31} \text{ kg}$$

$$m_{e^-} = 9.1 \times 10^{-31} \text{ kg}$$

To Find

Energy of the gamma ray photons $E = ?$

Formula $E = 2mc^2$

Solution

Using Einstein's mass-energy relation

$$E = 2 \times 9.1 \times 10^{-31} \text{ kg} \times (3 \times 10^8 \text{ m s}^{-1})^2$$

$$= 1.64 \times 10^{-13} \text{ J}$$

In eV

$$E = \frac{1.64 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} = 1.02 \times 10^6 \text{ eV} \\ = 1.02 \text{ MeV}$$

Result

$$E = 1.02 \text{ MeV}$$

Two gamma ray photons are emitted, each with 0.51 MeV, travelling in opposite directions to conserve momentum.

Short Questions

SQ 19.7.1 What is annihilation?

Annihilation

The event in which a particle and its antiparticle come close to each other and their mass is converted into energy, as gamma-ray photons.

SQ 19.7.2 Why are two gamma-ray photons emitted in the annihilation of an electron-positron pair?

Momentum conservation

The pair at rest has zero momentum, so the energy appears as two photons travelling in opposite directions, each of 0.51 MeV, whose momenta cancel.

SQ 19.7.3 Why are protons and antiprotons accelerated to nearly the speed of light before collision at the LHC?

Very high energies

Research on elementary particles needs very high energies, of the order of TeV, which recreate conditions similar to the early universe.

MCQ 19.7.1 Each photon produced by the annihilation of an electron-positron pair at rest has an energy of:

- | | |
|-------------|------------|
| A 0.51 MeV | B 1.02 MeV |
| C 0.255 MeV | D 2.04 MeV |

Solution: The pair gives 1.02 MeV, shared by two photons.

MCQ 19.7.2 The Higgs boson was found in:

- | | |
|-------------|--------------|
| A July 1932 | B June 2002 |
| C July 2012 | D March 2022 |

Solution: At CERN's Large Hadron Collider.

MCQ 19.7.3 The Large Hadron Collider is at:

- | | |
|--------|----------|
| A NIAB | B KANUPP |
| C CERN | D EAST |

Solution: It collides protons, antiprotons and heavy nuclei at TeV energies.

MCQ 19.7.4 The electro-weak force is mediated by the:

- | | |
|----------------------------------|-------------------------|
| A W^+ and W^- and Z bosons | B Higgs boson alone |
| C photons only | D top and bottom quarks |

Solution: They were found in high energy collisions as predicted.

Chapter Exercise

SQ 19.1 Define binding energy and how is it calculated?

Binding energy

The energy required to break the nucleus into its constituents, neutrons and protons, is called binding energy.

Calculation

First the mass defect is found, the mass by which the nucleus is lighter than its separate nucleons, and then it is converted to energy by Einstein's relation.

$$\Delta m = [Zm_p + (A - Z)m_n] - m_{\text{nucleus}}$$

$$\Delta E = (\Delta m)c^2$$

SQ 19.2 Comment on the statement. "In radioactive decay, if the probability of decay per unit time is doubled, its half-life will also be doubled."

The statement is wrong: the half-life will be halved.

Decay constant

The probability of decay per unit time is the decay constant λ , and the half-life is inversely proportional to it.

$$T_{1/2} = \frac{0.693}{\lambda}$$

So if λ is doubled, $T_{1/2}$ becomes half.

SQ 19.3 After four half-lives, what percentage of a radioactive sample remains?

Fraction left

Half of the sample decays in each half-life, so after four half-lives

$$N = \left(\frac{1}{2}\right)^4 N_0 = \frac{N_0}{16}$$

$$\frac{1}{16} \times 100\% = 6.25\%$$

6.25 percent of the sample remains.

SQ 19.4 Why natural uranium is said to be low grade nuclear fuel? Describe briefly.

Little fissile uranium

Naturally occurring uranium contains only 0.7 percent of $^{235}_{92}\text{U}$, the isotope that undergoes fission easily with slow neutrons; the rest is uranium-238.

Enrichment

So little fissile material cannot sustain the chain reaction, and for reactor fuel the quantity of $^{235}_{92}\text{U}$ is enriched to 2.4 to 3.0 percent.

SQ 19.5 Give an application of a radioactive tracers in medicine.

Thyroid gland

Radioactive iodine is absorbed mostly by the thyroid gland. A small quantity is given to the patient and a radiation detector measures how much the gland takes up: a diseased or hyperactive gland absorbs more than twice the amount of a normal gland. Iodine-131 is also used to combat cancer of the thyroid gland.

SQ 19.6 How is energy generated in the Sun? Describe briefly.

Nuclear fusion

In the core of the Sun the temperature is so high that light hydrogen nuclei move fast enough to overcome their electrostatic repulsion and fuse into helium.

Mass into energy

In each fusion some mass is lost, and its equivalent energy is given out.

$$\Delta E = (\Delta m)c^2$$

SQ 19.7 What is meant by the half-life of a radioactive material?

Half-life

The time taken for the activity of a sample to decrease to half of any starting value is called its half-life. In each half-life half of the nuclei present decay. The half-life of radon gas is 3.8 days and that of radium-226 is 1620 years.

$$T_{1/2} = \frac{0.693}{\lambda}$$

SQ 19.8 What is meant by controlled and uncontrolled fission chain reaction? Describe briefly.

Uncontrolled chain reaction

All the neutrons from each fission go on to cause further fissions, so the reaction proceeds so rapidly that a large amount of energy is released in an infinitesimally small time with a huge explosion, as in an atom bomb.

Controlled chain reaction

Only one neutron from each fission causes a further fission; the others escape or are absorbed by control rods. The reaction proceeds at its initial speed and releases energy at a steady rate, as in a nuclear reactor.

CRQ 19.1 In what ways a nuclear power station is different to a fossil fuel burning power station also called thermal power plant?

Source of heat

In a thermal power station coal or oil is burnt in a furnace to produce heat, while in a nuclear power station the reactor produces heat by fission. After that both work alike: the heat makes steam, the steam turns the turbine and the turbine turns the generator.

What the reactor needs

Fission releases much more energy than the burning of fossil fuels, so a little fuel runs the reactor for months.

The reactor needs a moderator, control rods, a coolant and a heat exchanger, and a concrete shield against radiation.

Its used fuel is intensely radioactive nuclear waste with half-lives of thousands of years, which must be stored safely.

CRQ 19.2 If fusion reactors are developed, what advantage are they likely to have over fission nuclear reactors?

More energy

Fusion gives more energy per nucleon than fission; the deuterium-tritium reaction alone releases 17.6 MeV.

Clean energy

The fuels are isotopes of hydrogen and the product is helium, whereas every fission fragment is radioactive. A fusion reactor would release clean energy, like the Sun, without the long-lived radioactive waste that a fission reactor leaves.

CRQ 19.3 The activity of a radioactive sample is found to decay by 10 percent in a year. What will be the activity after another year? Explain.

The activity after another year will be 81 percent of the original.

Same fraction each year

Radioactive decay removes the same fraction of the remaining nuclei in each equal interval, because the number decaying is proportional to the number present. After the first year

$$A_1 = 0.9A_0$$

In the second year 10 percent of this remaining activity decays.

$$A_2 = 0.9 \times 0.9A_0 = 0.81A_0$$

So it falls by 9 percent of the original activity, not by another 10 percent.

CRQ 19.4 A source has an activity of 10×10^4 Bq and a half-life of 20 minutes. What will be the activity of the source after one hour?

Number of half-lives

One hour is three half-lives of 20 minutes.

$$n = \frac{60 \text{ min}}{20 \text{ min}} = 3$$

Activity left

The activity halves in each half-life.

$$A = \left(\frac{1}{2}\right)^3 \times 10 \times 10^4 \text{ Bq} = 1.25 \times 10^4 \text{ Bq}$$

CRQ 19.5 What factors make a fusion reaction difficult to achieve?

Electrostatic repulsion

The two nuclei are both positively charged, and work has to be done against the repulsive force between them to bring them very close together.

Very high temperature

The nuclei need very high speeds, which a temperature of about 15 million kelvin gives. Such a temperature cannot ordinarily be achieved, and so far fusion has not been brought under control like fission.

CRQ 19.6 In what way a nuclear reaction is different from chemical reaction? Explain.

Nucleus and electrons

A chemical reaction rearranges the electrons of atoms and the elements stay the same. A nuclear reaction changes the nucleus itself, so one element is transformed into others, as uranium into barium and krypton.

Energy

A nuclear reaction converts a measurable mass into energy, about 200 MeV in the fission of one uranium nucleus, which is very much more than a chemical reaction such as the burning of fossil fuel releases.

CRQ 19.7 Suggest a reason why neutrons are absorbed in the boron rods and the rods become hot as a result of this reaction.

Neutron absorption

Boron nuclei have the property of absorbing neutrons, so a neutron that reaches a rod is captured in a nuclear reaction and removed from the chain reaction.

Heating

The absorption reaction gives out energy, and the moving products together with the kinetic energy of the absorbed neutrons collide with the atoms of the rod, so the energy becomes heat and the rods get hot.

MCQ 19.1 What is the half-life of a radio nuclide if 1/16 of its mass is present after 2h?

A	15 min.	B	30 min.
C	45 min.	D	60 min.

Solution: $1/16$ is $(1/2)^4$, so four half-lives pass in 2 h and each is 30 min.

MCQ 19.2 Gamma radiations are emitted due to:

- | | |
|----------------------------------|------------------------|
| A de-excitation of atoms | B excitation of atoms |
| C de-excitation of nuclei | D excitation of nuclei |

Solution: An excited nucleus falls to a lower nuclear energy level and gives out the difference as a gamma photon.

MCQ 19.3 Cadmium rods are used in nuclear reactor for:

- | | |
|-----------------------------|---------------------------------|
| A slowing down fast neutron | B speeding up slow neutron |
| C absorbing neutrons | D starting the nuclear reaction |

Solution: Cadmium and boron absorb neutrons, so moving the rods in or out controls the chain reaction.

MCQ 19.4 The mass of fissionable material needed for self-sustaining chain reaction is called:

- | | |
|------------------------|---------------------|
| A supercritical mass | B fermi mass |
| C critical mass | D sub-critical mass |

Solution: The minimum mass that sustains a nuclear chain reaction is the critical mass.

MCQ 19.5 Graphite and heavy water are two common moderators used in a nuclear reactor. The function of the moderator is:

- | | |
|--|--|
| A to slow down the neutrons to thermal energies | B to absorb the neutrons and stop the chain reaction |
| C to cool the reactor | D to control the energy released in the reactor |

Solution: Fast fission neutrons are not suitable for fission; the moderator slows them down.

MCQ 19.6 The mass of a nucleus is always:

- | | |
|--|---|
| A an integral number of proton masses | B equal to the mass of all other nuclei of the same element |
| C equal to the sum of the masses of proton and neutron | D less than the sum of masses of the constituent particles |

Solution: The missing mass, the mass defect, has become the binding energy of the nucleus.

MCQ 19.7 Binding energy per nucleon is:

- | | |
|----------------------------|---|
| A greater for heavy nuclei | B least for heavy nuclei |
| C greater for light nuclei | D greater for medium weight nuclei |

Solution: The curve peaks at about 8.7 MeV near iron, between mass numbers 50 and 80.

MCQ 19.8 What remains unchanged in a fusion event?

- | | |
|-----------------------------|--------------------|
| A Energy | B Mass of nucleons |
| C Number of nucleons | D Temperature |

Solution: The mass numbers on the two sides of a fusion equation add up to the same total, while some mass becomes energy.

MCQ 19.9 Radioactive tracers are also employed to follow the path that various chemicals or food constituents take in:

- | | |
|----------------|-----------------------|
| A human bodies | B animals |
| C plants | D all of these |

Solution: Tracers follow chemical and biological processes in the human body, in animals and in plants.

MCQ 19.10 The K.E. of the fission fragments is ultimately converted to:

- | | |
|--------------------|----------------------|
| A potential energy | B heat energy |
| C magnetic energy | D chemical energy |

Solution: The fast fragments collide with the atoms of the fuel and their kinetic energy becomes heat.

Numerical 19.1 Find the mass defect and binding energy of the deuterium. The mass of deuterium is 3.3435×10^{-27} kg. (mass of proton $m_p = 1.6726 \times 10^{-27}$ kg, mass of neutron $m_n = 1.6749 \times 10^{-27}$ kg) (Ans: 2.23 MeV)

Given Data

$$m_{\text{nucleus}} = 3.3435 \times 10^{-27} \text{ kg}$$

$$m_p = 1.6726 \times 10^{-27} \text{ kg}$$

$$m_n = 1.6749 \times 10^{-27} \text{ kg}$$

To Find

Mass defect $\Delta m = ?$

Binding energy B. E = ?

Formula

$$\Delta m = m_p + m_n - m_{\text{nucleus}}$$

$$\text{B. E} = \Delta m c^2$$

Solution

The mass defect

$$\begin{aligned} \Delta m &= (1.6726 + 1.6749 - 3.3435) \times 10^{-27} \text{ kg} \\ &= 0.0040 \times 10^{-27} \text{ kg} \end{aligned}$$

The binding energy

$$\begin{aligned} \text{B. E} &= 0.0040 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m s}^{-1})^2 \\ &= 3.6 \times 10^{-13} \text{ J} \end{aligned}$$

In electron volts

$$\begin{aligned} \text{B. E} &= \frac{3.6 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 2.25 \times 10^6 \text{ eV} \end{aligned}$$

Result

$$\Delta m = 4.0 \times 10^{-30} \text{ kg}$$

$$\text{B. E} = 2.25 \text{ MeV}$$

Note

The book prints 2.23 MeV; its own data give 2.25 MeV.

Numerical 19.2 The decay constant of strontium is $1.99 \times 10^{-5} \text{ s}^{-1}$. Find its half-life. (Ans: 9.7 hours)

Given Data $\lambda = 1.99 \times 10^{-5} \text{ s}^{-1}$

To Find Half-life $T_{1/2} = ?$

Formula $T_{1/2} = \frac{0.693}{\lambda}$

Solution

Putting the values in the formula

$$\begin{aligned} T_{1/2} &= \frac{0.693}{1.99 \times 10^{-5} \text{ s}^{-1}} \\ &= 3.48 \times 10^4 \text{ s} \end{aligned}$$

In hours

$$T_{1/2} = \frac{3.48 \times 10^4}{3600} \text{ h} = 9.7 \text{ h}$$

Result

$$T_{1/2} = 9.7 \text{ hours}$$

Numerical 19.3 Half-life of krypton is 3.16 minutes. Out of 40 g of krypton, how much will be left after 12.64 minutes? (Ans: 2.5 g)

Given Data

$$T_{1/2} = 3.16 \text{ min}$$

$$N_o = 40 \text{ g}$$

$$t = 12.64 \text{ min}$$

To Find Mass left $N = ?$

Formula $n = \frac{t}{T_{1/2}}$

$$N = \left(\frac{1}{2}\right)^n N_o$$

Solution

Number of half-lives

$$n = \frac{12.64 \text{ min}}{3.16 \text{ min}} = 4$$

Mass left

$$N = \left(\frac{1}{2}\right)^4 \times 40 \text{ g} = 2.5 \text{ g}$$

Result $N = 2.5 \text{ g}$

Numerical 19.4 A 2048 g ($\approx 2 \text{ kg}$) sample of a radioactive material decreases to 32 g in 18 years. What is its half-life? (Ans: 3 years)

Given Data

$$N_o = 2048 \text{ g}$$

$$N = 32 \text{ g}$$

$$t = 18 \text{ years}$$

To Find Half-life $T_{1/2} = ?$

$$\text{Formula } N = \left(\frac{1}{2}\right)^n N_o$$

$$T_{1/2} = \frac{t}{n}$$

Solution

The fraction left

$$\frac{N}{N_o} = \frac{32}{2048} = \frac{1}{64} = \left(\frac{1}{2}\right)^6$$

So six half-lives have passed.

$$n = 6$$

Half-life

$$T_{1/2} = \frac{18 \text{ years}}{6} = 3 \text{ years}$$

Result $T_{1/2} = 3 \text{ years}$

Numerical 19.5 An alpha particle has an energy of 5.56 MeV when released from the Radium atom. Determine its velocity. (Using $m_\alpha = 6.64 \times 10^{-27} \text{ kg}$). (Ans: $1.67 \times 10^7 \text{ m s}^{-1}$)

Given Data

$$E = 5.56 \text{ MeV} = 5.56 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ = 8.90 \times 10^{-13} \text{ J}$$

$$m_\alpha = 6.64 \times 10^{-27} \text{ kg}$$

To Find Velocity $v = ?$

$$\text{Formula } E = \frac{1}{2} m_\alpha v^2$$

$$v = \sqrt{\frac{2E}{m_\alpha}}$$

Solution

Putting the values in the formula

$$v = \sqrt{\frac{2 \times 8.90 \times 10^{-13} \text{ J}}{6.64 \times 10^{-27} \text{ kg}}} \\ = 1.64 \times 10^7 \text{ m s}^{-1}$$

Result $v = 1.64 \times 10^7 \text{ m s}^{-1}$ **Note**

The book prints $1.67 \times 10^7 \text{ m s}^{-1}$; its own data give $1.64 \times 10^7 \text{ m s}^{-1}$.

Numerical 19.6 Find the energy released during the nuclear reaction ${}^7_3\text{Li} + {}^1_1\text{H} \rightarrow 2 {}^4_2\text{He}$. Mass of ${}^4_2\text{He}$ is 4.00388 u, of ${}^1_1\text{H}$ is 1.00814 u and of ${}^7_3\text{Li}$ is 7.01823 u. (Ans: 17.3 MeV)

Given Data

$$m_{Li} = 7.01823 \text{ u}$$

$$m_H = 1.00814 \text{ u}$$

$$m_{He} = 4.00388 \text{ u}$$

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

To FindEnergy released $\Delta E = ?$ **Formula**

$$\Delta m = (m_{Li} + m_H) - 2m_{He}$$

$$\Delta E = (\Delta m)c^2$$

Solution

The mass lost

$$\Delta m = (7.01823 + 1.00814) \text{ u} - 2 \times 4.00388 \text{ u} \\ = 0.01861 \text{ u}$$

In kilograms

$$\Delta m = 0.01861 \times 1.66 \times 10^{-27} \text{ kg} \\ = 3.09 \times 10^{-29} \text{ kg}$$

The energy released

$$\Delta E = 3.09 \times 10^{-29} \text{ kg} \times (3 \times 10^8 \text{ m s}^{-1})^2 \\ = 2.78 \times 10^{-12} \text{ J}$$

In electron volts

$$\Delta E = \frac{2.78 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \\ = 17.4 \times 10^6 \text{ eV}$$

Result $\Delta E = 17.4 \text{ MeV}$ **Note**

The book prints 17.3 MeV, which is what $1 \text{ u} = 931 \text{ MeV}$ gives; with $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$ its data give 17.4 MeV.

Self Test

Seventeen questions across the whole chapter. Work through them without the notes, then check the answers keyed at the foot of the page.

Q 1 The missing mass of a nucleus is converted into its:

A	kinetic energy	B	electric charge
C	binding energy	D	atomic number

Q 2 The number of nucleons in $^{56}_{26}\text{Fe}$ is:

A	26	B	30
C	82	D	56

Q 3 The binding energy of the helium nucleus is about:

A	8.7 MeV	B	27 MeV
C	2.2 MeV	D	200 MeV

Q 4 The binding energy per nucleon of uranium is about:

A	7.6 MeV	B	8.7 MeV
C	1.0 MeV	D	17.6 MeV

Q 5 When uranium-235 absorbs a slow neutron, the compound nucleus formed is:

A	$^{238}_{92}\text{U}$	B	$^{239}_{93}\text{Np}$
C	$^{236}_{92}\text{U}$	D	$^{141}_{56}\text{Ba}$

Q 6 In a controlled chain reaction the number of neutrons from each fission that cause a further fission is, on average:

A	two	B	one
C	three	D	none

Q 7 In a nuclear power plant the steam that drives the turbine is made in the:

A	pressurizer	B	pump
C	control rods	D	heat exchanger

Q 8 Uranium-239 changes into neptunium-239 with a half-life of:

A	23.5 minutes	B	2.4 days
C	24,100 years	D	3.8 days

Q 9 Two light nuclei can fuse only if work is done against their:

A	electrostatic repulsion	B	gravitational attraction
C	magnetic attraction	D	binding energy

Q 10 When two deuterons fuse to form tritium, the other particle produced is a:

A	neutron	B	proton
C	positron	D	helium nucleus

Q 11 Fusion takes place naturally in the:

A	core of the Earth	B	fuel rods of a reactor
C	core of the stars	D	upper atmosphere

Q 12 The activity of a radioactive sample is directly proportional to the:

A	half-life	B	mass number
C	temperature of the sample	D	number of undecayed nuclei

Q 13 The number of atoms left from N_o after n half-lives is:

A	$\frac{N_o}{2n}$	B	$N_o \left(1 - \frac{1}{2^n}\right)$
C	$\left(\frac{1}{2}\right)^n N_o$	D	$2^n N_o$

Q 14 A source has a half-life of 2 hours. Its activity falls to one quarter in:

A	2 hours	B	4 hours
C	8 hours	D	1 hour

Q 15 Inside a living organism a radioactive isotope behaves:

A	faster than the normal isotope	B	as an inert gas
C	only as a source of heat	D	in the same way as the normal isotope

Q 16 In the annihilation of a particle and its antiparticle, the quantities conserved are:

A	mass-energy and momentum	B	mass only
C	charge and mass only	D	kinetic energy only

Q 17 The total energy given out when an electron and a positron at rest annihilate is about:

A	0.51 MeV	B	17.6 MeV
C	1.02 MeV	D	200 MeV

Answers 1 (c) 2 (d) 3 (b) 4 (a) 5 (c) 6 (b) 7 (d) 8 (a) 9 (a) 10 (b) 11 (c) 12 (d) 13 (c) 14 (b) 15 (d) 16 (a) 17 (c)